Practice Problems

1. Suppose $E[X \mid Y] = 18 - \frac{3Y}{5}$ and $E[Y \mid X] = 10 - \frac{X}{5}$. Compute $E[X]$ and $E[Y]$.

2. Consider the Markov chain with the following probability transition matrix

$$P = \begin{bmatrix}
0.0 & 0.0 & 1.0 \\
1.0 & 0.0 & 0.0 \\
0.5 & 0.5 & 0.0
\end{bmatrix}$$

(a) Is the Markov chain irreducible?
(b) Is the Markov chain periodic? If it is, what is the period?
(c) Compute the steady state probabilities.

3. Consider a Markov chain with probability transition matrix

$$P = \begin{bmatrix}
0.3 & x & 0.5 \\
0.6 & 0.2 & y \\
a & b & c
\end{bmatrix}$$

(a) Find $x$ and $y$.
(b) Find $a$, $b$ and $c$ so that $\pi_1 = \pi_2 = \pi_3$ where $\pi_1$, $\pi_2$ and $\pi_3$ are the steady state probabilities.

4. Suppose that $X$ is an exponential random variable with mean $1/\lambda$. The parameter $\lambda$ itself is a random variable whose distribution is exponential with mean 1. Compute $P\{X \leq 10\}$.

Consider the following problems from chapter 4 of your book
1, 2, 8, 17, 18, 23, 25, 34