Name

Please be neat and show all your work so that I can give you partial credit.
GOOD LUCK.

Question 1
Question 2
Question 3
Total
1.a Consider a stable $G/G/1$ queue. Prove or disprove that the long run distribution of the system size prior to arrivals is equal to the long run distribution of system size after the departures.

b. Consider a stable $G/M/1$ queue. Assume that the interarrival times are deterministic and equal to $1/\lambda$. Compute $\beta(z)$ for this system where as defined in class

$$\beta(z) = \sum_{n=0}^{\infty} z^n \int_0^{\infty} \frac{e^{-\mu t} (\mu t)^n}{n!} dG(t).$$

with $G(\cdot)$ being the interarrival time distribution.
Consider a closed Jackson network with \( K \) stations and \( N \) customers in the network. Assume that there is only one server at each station \( i \) with service rate \( \mu_i \). Show that the stationary probability distribution has the following form

\[
p_{n_1, n_2, \ldots, n_K} = C \prod_{i=1}^{K} \rho_i^{n_i}
\]

where

\[
C = \left( \sum_{n_1 + n_2 + \ldots + n_K = N} \prod_{i=1}^{K} \rho_i^{n_i} \right)^{-1},
\]

and \( \rho_i = \frac{\lambda_i}{\mu_i} \).
(30) 3. What is the expected length of the busy period in a stable $M/G/1$ queue with arrival rate $\lambda$ and service rate $\mu$. 