Homework 1
Due September 13, Tuesday

1. Consider an $M/M/1$ queue with $\lambda < \mu$. Compute the stationary distribution of the number of customers in the system
   a. at the time of departures
   b. at the time of arrivals
   c. at instants of time at which the number of customers in the system changes (i.e., consider the embedded Markov chain at the instants at which the number of customers in the system changes, defining the state to be the number of customers in the system immediately following the state change)

2. Obtain the Laplace-Stieltjes transform for the steady state waiting time and the steady state time spent in the system for an $M/M/1$ queue. That is, compute $E[e^{-sT}]$ and $E[e^{-sT_q}]$ for $s \geq 0$. Do your results agree with the expressions derived in class?

3. Show that the distribution of an arbitrary interdeparture time for the $M/M/1$ queue in steady state is exponential with the same parameter as the interarrival time distribution. Also argue that the interdeparture times are independent of each other. Hence, the departure process is a Poisson process with the same rate as the arrival process. (This is Burke’s theorem)