Homework 1
Due September 15, Thursday

1. Consider a generalization of the inventory problem that we discussed in class. Suppose that the unfilled orders may be backlogged indefinitely with a cost of $b(u)$ units if $u$ units are backlogged for one period. Assume revenue is received at the end of the period in which orders are placed and that backlogging costs are charged only if a unit is backlogged for an entire month, in which case backlogging cost is incurred at the beginning of that month. Identify the state and action spaces and derive the transition probabilities and expected rewards for this model.

2. Consider the equipment replacement problem that we discussed in class with $N = 3$. Let $p(j) = \pi^j(1 - \pi)$ for $j = 0, 1, 2, \ldots$ with $\pi = 0.4$, $R = 0$, $K = 5$, $h(s) = 2s$ and $r_3(s) = \max\{5 - s, 0\}$. Compute the optimal replacement policy for this three period problem.

3. Suppose we have $N$ stages to construct $I$ successful components sequentially. At each stage we allocate a certain amount of money for the construction of a component. If $y$ ($y \in [0,Y]$) is the amount allocated, then the component constructed will be a success with probability $P(y)$, where $P$ is a continuous nondecreasing function satisfying $P(0) = 0$. After each component is constructed, we are informed whether or not it is successful. If, at the end of the $N$ stages, we are $j$ components short, then a final penalty cost $C(j)$ is incurred. Formulate this problem as a finite horizon Markov Decision Process problem. Write down the optimality equations. Does there exist a Markovian Deterministic policy which is optimal? Justify your answer.