Solutions to Homework 2

Problem 1

Data: $c_s = 50$, $p = 20$, $c_h = 2$, and $D \sim \text{Unif}(500, 1500)$

(a) Optimum amount of bananas to order each week is $Q^*$ such that

$$F(Q^*) = \left( \frac{c_s - p}{c_s + c_h} \right) = \left( \frac{50 - 20}{50 + 2} \right) = 0.5769$$

$$F(Q^*) = \left( \frac{Q^* - 5000}{1000} \right) = 0.5769, \quad Q^* \approx 1077.$$

(b) Suppose $Q = 1077$. Then

$$E[\text{Profit}(1077, D)] = 50E[\min(1077, D)] - 2E[\max(1077 - D, 0)] - 20*1077.$$  

Note that

$$E[\min(1077, D)] = \int_{500}^{1500} \min(1077, x) \frac{1}{1000} dx$$

$$= \frac{1}{1000} \int_{500}^{1077} xd\ x + \frac{1}{1000} \int_{1077}^{1500} 1077 dx = 910.53, \quad \text{and}$$

$$E[1077 - D]^+ = \int_{500}^{1077} \frac{1}{1000} (1077 - x) dx = 166.46$$

So that $E[\text{profit}(1077, D)] = 50*910.53 - 2*166.46 - 20*1077 = 23653.78$ cents.

Alternatively,

$$E[\text{profit}(1077, D)] = (50 - 20)E[\min(1077, D)] - [20 - (-2)]E[1077 - D]^+$$

$$= 30*910.53 - 22*166.46$$

$$= 23653.78 \text{ cents}$$

(c) Use the same approach as in part (b), except now $F$ is an exponential distribution function with mean 1000 so that $F(Q^*) = 0.5769$ and
\[ 1 - e^{-\frac{1}{1000}Q'} = 0.5769 \Rightarrow Q' \approx 860 \]

(with a corresponding expected profit,

\[ E[\text{profit}(860, D)] = 30E[\text{min}(860, D)] - 22E[860 - D]^+ \]

\[ E[\text{min}(860, D)] = \int_{0}^{\infty} \text{min}(860, x) \frac{1}{1000} e^{-x/1000} dx \]

\[ = \frac{1}{1000} \int_{0}^{860} xe^{-x/1000} dx + \frac{1}{1000} \int_{860}^{\infty} 860e^{-x/1000} dx = 212.92 + 364.92 = 576.84 \]

\[ E[860, D]^+ = \int_{0}^{860} \frac{1}{1000} (860 - x)^+ e^{-x/1000} dx = 283.16 \]

and thus, \( E[\text{profit}(860, D)] = 11075.68 \) cents.)

**Problem 2**

c\(_s\) = $20, p = $5, and c\(_h\) = $0.

(a) The mean demand is

\[ E[D] = 10(.1) + 11(.4) + 12(.2) + 13(.2) + 14(.1) = 11.8 \]

(b) Suppose \( q = 12 \). Then

\[ E[\text{Profit}(12,D)] = 20E[\text{min}(12,D)] + 0E[\text{max}(12-D,0)] - 5*12 \]

\[ = 20[10(.1)+11(.4)+12(.2)+12(.2)+12(.1)] + 0 - 60 \]

\[ = 168 \]

Alternatively,

\[ E[\text{profit}(12,D)] = (20-5)E[D] - 5E[D]^- \]

\[ = 15[10(.1) + 11(.4) + 12(.5)] - 5[1(.4) + 2(.1)] = 168 \]

(c) Here

\[ E[\text{cost}] = pE[D] - E[\text{profit}] = 236 - 168 = 68 \]

(d) The optimal amount is the smallest value \( Q^* \) that satisfies

\[ F(Q^*) = \left( \frac{c_s - p}{c_s + c_h} \right) = \left( \frac{20 - 5}{20 + 0} \right) = \frac{3}{4}. \]
Hence $Q^* = 13$.

(e) Since $c_h = 0$, it follows that the expected overstock cost is zero.

(f) This is similar to (d), except now $F$ is a normal distribution function (continuous demand distribution).

Since $D \sim N(1000,200)$, $Z = \frac{D - 1000}{\sqrt{200}} \sim N(0,1^2)$. Thus, we would like to find $Q^*$ satisfying

$$F(Q^*) = P(D \leq Q^*) = P\left(\frac{D - 1000}{\sqrt{200}} \leq \frac{Q^* - 1000}{\sqrt{200}}\right) = P\left(Z \leq \frac{Q^* - 1000}{\sqrt{200}}\right) = \frac{3}{4}.$$ 

After looking up the value from the standard normal table, we can find that $Q^* = 1009.62$.

**Problem 3**

In this case, $c_s =$per-unit shortage cost=$640 + 50 = $690, $c_h =$25 and $p =$380.

(a) Assume 13 cameras are ordered at the beginning of the month. Let $E[U]$ and $E[O]$ denote the expected understock cost and the expected overstock cost, respectively.

$$E[U] = 690E[D - 13]^+ = 690 \left(1 + \frac{2 + 3}{7}\right) = 591.43$$

$$E[O] = 25E[13 - D]^+ = 25 \left(1 + \frac{2 + 3}{7}\right) = 21.43$$


(b) We are interested in the smallest $Q^*$ that satisfies

$$F(Q^*) \geq \frac{c_s - p}{c_s + c_h} = \frac{690 - 380}{690 + 25} = 0.4335.$$ 

This value is $Q^* = 13$. 