ISyE 6656 Final Exam
Spring 2003

Name

Please be neat and show all your work so that I can give you partial credit. GOOD LUCK AND HAVE A GREAT SUMMER.

Question 1
Question 2
Question 3
Question 4
Question 5

Total
(20) 1. Consider an M/G/1 queue with the following variation. The first customer in a busy period has service time distribution $G_1$ with expected value $E[S_1]$ and all others have service time distribution $G_2$ with expected value $E[S_2]$. Assume that the arrival rate is $\lambda$ and $\rho = \lambda E[S_2] < 1$

a. What is the expected length of the busy period?

b. What is the expected number of customers served in a busy period?
2. Consider a stable $G/M/1$ queue with arrival rate $\lambda$ and service rate $\mu$. Assume that the interarrival times have Erlang-2 distribution i.e. $f(t) = 2\lambda(2\lambda t)e^{-2\lambda t}$ for $t \geq 0$. Let $N_a$ denote the number of customers found in the system at the time of an arrival in the long-run. Prove or disprove the following statement

$$P(N_a > 0) < \rho.$$  

(Hint: You may want to explore whether Erlang-2 has increasing failure rate or decreasing failure rate).
3. Consider a closed queueing network consisting of two customers moving among two servers, and suppose that after each service completion the customer is equally likely to go to either server, i.e. \( p_{12} = p_{21} = 1/2 \). Assume that server \( i \) has exponentially distributed service time with rate \( \mu_i \), \( i = 1, 2 \). Determine the stationary distribution and the expected number of customers at each server in the long run.
(20) **4.** Show that in a stable $G/M/1$ queue, stationary time spent in the system has the exponential distribution.

**b.** Consider two $G/M/1$ queues with service rate $\mu$ and $\rho_i < 1$, where $A_2 \leq A_1$. Show that the stationary number of customers in the system are ordered in standard stochastic ordering for these two systems. Note that $A_i$ denotes the distribution of interarrival times for system $i$, $i = 1, 2$. 
Consider a stable $M/G/1$ queue. Show that the expected amount of time required to finish the item in service encountered by an arbitrary arrival in steady state is given by

$$\frac{\lambda}{2} E[S^2]$$

where $\lambda$ is the arrival rate and $S$ is the total service time of an item.