Name

Please be neat and show all your work so that I can give you partial credit. GOOD LUCK.
1. Customers arrive according to a Poisson process with mean interarrival time $1/\lambda$. There are $s$ servers in parallel and the service times are mutually independent exponentially distributed random variables with common mean $1/\mu$. At time 0 all $s$ servers are occupied and no customers are waiting.

(a) Find the probability that the next arriving customer finds all the servers busy.

(b) Let $N$ be the number of customers who arrive prior to the first service completion. Find $P\{N = j\}$ for $j = 0, 1, 2, \ldots$.

(c) Find the probability that the next arriving customer finds at least 2 idle servers.
2. A customer makes deposits at a bank according to a Poisson process of rate 1 per week. The size of the deposit is a random variable that is uniformly distributed over [100,200] dollars. Successive deposits are independent identically distributed random variables. Unknown to him the customer’s spouse makes withdrawals from the same account according to an independent Poisson process with rate 2 per week. The successive amounts withdrawn are independent identically distributed random variables that are uniformly distributed over [50,100] dollars. Let \( Z(t) \) be the account balance at time \( t \). Assuming that the customer has unlimited credit line, compute the mean and variance of \( Z(t) \).
3. (30) Consider a system consisting of $n$ components. The lifetime of the $i$th component is an exponential random variable with rate $\lambda_i$. As soon as any one of the components fails the system fails. The repair time of the $i$th component is exponential with rate $\mu_i$. When the system is down, no more component failures take place. Compute the long run probability that the system is working.