Name

Please be neat and show all your work so that I can give you partial credit. GOOD LUCK.

Question 1
Question 2
Question 3

Total
(30) 1. Consider a single-server model with Poisson input and an average customer arrival rate of 30/hr. Currently, service is provided by a mechanism that takes exactly 1.5 min.

(a) Is the current system stable? Justify your answer.

(b) Now suppose that the system can be served instead by a mechanism that has an exponential service time distribution. What must be the mean service time of this mechanism to ensure the same long run average waiting time in the system?
2. The arrival process of customers at a taxi stand is Poisson with rate \( \lambda \) and the arrival process of the of taxis at the stand is Poisson with rate \( \mu \). Arriving customers who find taxis waiting leave immediately in a taxi. Arriving taxis who find customers waiting leave immediately with one customer each. Otherwise, the customers will queue up to a limit of \( c \) customers, i.e., arriving customers who find \( c \) other customers waiting leave immediately without a taxi. Similarly, taxis queue up to a limit of \( t \) taxis.

(a) (15) Define a suitable continuous time Markov chain for this problem

(b) (10) Write down the balance equations. When does the stationary distribution exist?

(c) (15) In terms of the long run probabilities, what is the average number of customers and average number of taxis at the stand?
3. (a) Consider an $M/M/1$ queue with $\lambda < \mu$. You do not know the exact service discipline but the server is not allowed to idle unless the system is empty. Can you still determine the long-run average waiting time in the system? If you can, what is it?

(b) What about the distribution of the stationary waiting time in the system?