ISyE 6650 Exam # 2
Fall 2002

Name

Please be neat and show all your work so that I can give you partial credit. GOOD LUCK.
(30) 1. John is married to Mary. Let $H_n$ be the number of happy persons in this marriage on day $n$. Assume that the probability that John is happy on day $n$ is $(1 + H_{n-1})/4$. The probability that Mary is happy on day $n$ is also $(1 + H_{n-1})/4$. What is the long-run probability that they are both happy?
(40) 2. (a) Sue and Liz arrive at a beauty salon together and plan to leave together. Sue needs a perm, and Liz a manicure. The duration of a perm is exponentially distributed with rate $\lambda$; that of a manicure is exponentially distributed with rate $\mu$. If both are served immediately, what is expected duration one has to wait for the other?

(b) Let $X$ and $Y$ be two independent exponentially distributed random variables with rate $\lambda$ and $\mu$ respectively. Compute $E[X|X < Y]$ and $E[X|X > Y]$. 

Consider a queueing system with Poisson arrivals and exponential service times. Suppose that when there are $n$ customers in the system the arrival rate is $(n + 1)\lambda$ and the service rate is $n^2\mu$.

(a) Compute the long-run distribution of the number of customers in the system. Make sure to specify when this distribution exists.

(b) Compute the expected number of customers in the system in the long run.