Name

This is a closed notes and closed book exam. Please be neat and show all your work so that I can give you partial credit. GOOD LUCK AND HAVE A GREAT SUMMER!
1. A doctor has scheduled two appointments, one at 1 p.m. and the other at 1:30 p.m. The amounts of time that appointments last are independent exponential random variables with mean 30 minutes. Assuming that both patients are on time, find the expected amount of time that the 1:30 appointment spends at the doctor’s office.
Customers arrive at a service facility according to a Poisson process with rate 40 per hour. When two or fewer customers are present, a single attendant operates the facility, and the service time for each customer is exponentially distributed with a mean value of two minutes. However, when there are three or more customers at the facility, the attendant is joined by an assistant and working together, they reduce the mean service time to one minute. Assume that the system can hold at most four customers.

a. What is the long run proportion of time that both servers are free?

b. What is the expected number of customers in the system in the long run?

c. What is the expected time spent in the system in the long-run?
(20) 3. Suppose that coin 1 has probability 0.7 of coming up heads, coin 2 has probability 0.6 of coming up heads. If the coin flipped today comes up heads, then we select coin 1 to flip tomorrow, and if it comes up tails, then we select coin 2 to flip tomorrow. If the coin initially flipped is equally likely to be coin 1 or coin 2, then what is the probability that the coin flipped on the second day after the initial flip is coin 1?
4. Suppose that during a football game, lemonade sells for $15 per gallon but only costs $2 per gallon to make. If they run out of lemonade during the game, it will be impossible to get more. On the other hand, leftover lemonade has a negligible value. Assume that you believe the fans would buy 10 gallons with probability 3/10, 11 gallons with probability 4/10, 12 gallons with probability 2/10, 13 gallons with probability 5/100, and 14 gallons with probability 5/100.

a. What is the mean demand?

b. What is the optimal amount of lemonade to be prepared for a game?

c. If the optimal amount is prepared for a game, what is the expected loss from overstock, i.e., the expected holding cost?
5. Consider a service facility with Poisson arrivals and exponential service times and a single server. However, assume that a customer waiting in the queue can get impatient and leave the system (without being served) after an exponential amount of time with rate 20 per hour. The arrival rate is 30 per hour and the service rate is 10 per hour. Assume that the system has infinite capacity.

a. Does the stationary distribution of the number of customers in the system exist? Justify your answer. If it does, compute the stationary distribution.

b. What is the expected number of customers in the system in the long run?