Problem 1
Nocedal and Wright, Problem 5.1

Problem 2
Nocedal and Wright, Problem 5.4

Problem 3
Nocedal and Wright, Problem 5.11

Problem 4
Consider a symmetric positive definite matrix $Q \in \mathbb{R}^{n\times n}$, and the associated norm $\|x\|_Q := \sqrt{x^T Q x}$. Consider $Q$-conjugate directions $d_0, d_1, \ldots, d_{n-1} \in \mathbb{R}^n$ generated from linearly independent vectors $p_0, p_1, \ldots, p_{n-1} \in \mathbb{R}^n$. Show that, for each $k = 1, \ldots, n - 1$, $d_k = p_k - \hat{p}_k$, where $\hat{p}_k$ is the projection of $p_k$ onto the subspace spanned by $p_0, \ldots, p_{k-1}$ (or the subspace spanned by $d_0, \ldots, d_{k-1}$) with respect to the $\| \cdot \|_Q$-norm, that is,

$$\hat{p}_k = \arg\min \{ \|p_k - p\|_Q : p \in [p_0, \ldots, p_{k-1}] \}$$

That is, $d_k$ is the part of $p_k$ that remains after we subtract the projection of $p_k$ onto the subspace spanned by $p_0, \ldots, p_{k-1}$. 