ISyE 6663 Optimization III

Spring 2003

Assignment 2

Issued: January 23, 2003

Due: January 30, 2003

Problem 1
Consider a quadratic function $f : \mathbb{R}^n \mapsto \mathbb{R}$ given by $f(x) := \frac{1}{2}x^T Gx + d^T x$, where $G \in \mathbb{R}^{n \times n}$ and $d \in \mathbb{R}^n$.

(1) Show that we can assume, without loss of generality, that G is symmetric.

(2) Verify that $\nabla f(x) = Gx + d$.

(3) Show that f is Lipschitz continuously differentiable, that is, there is a constant L such that

$$\|\nabla f(x) - \nabla f(y)\|_2 \quad \leq \quad L\|x - y\|_2$$

for all $x, y \in \mathbb{R}^n$.

(4) What is the smallest constant L such that the Lipschitz inequality above holds?

Problem 2
Nocedal and Wright, Problem 2.5

Problem 3
Nocedal and Wright, Problem 2.6

Problem 4
Nocedal and Wright, Problem 2.7

Problem 5
Nocedal and Wright, Problem 2.9

Problem 6
Nocedal and Wright, Problem 2.10

Problem 7

Consider a quadratic function $f : \mathbb{R}^2 \mapsto \mathbb{R}$ given by $f(x, y) := ax^2 + by^2 + cxy + dx + ey$.

(1) Suppose that $a = b = 1$. Identify and classify the stationary points of f (it may depend on the other parameters c, d, e).

(2) Identify and classify the stationary points of f as a function of all the parameters a, b, c, d, e).

Problem 8

A local minimizer in every direction is not necessarily a local minimum:
Consider a differentiable function $f : \mathbb{R}^n \mapsto \mathbb{R}$. Consider a point x^* that is a local minimizer of f along every line through x^*; that is, the function $g(\alpha) := f(x^* + \alpha d)$ is minimized at $\alpha = 0$ for all $d \in \mathbb{R}^n$.

(1) Show that $\nabla f(x^*) = 0$.

(2) Consider the function $f : \mathbb{R}^2 \mapsto \mathbb{R}$ given by $f(x, y) := (x - ay^2)(x - by^2)$ for some $a, b > 0$, $a \neq b$. Show that $x^* = 0$ is a local minimizer of f along every line through x^*, but that x^* is not a local minimum of f.

Problem 9

Nocedal and Wright, Problem 2.12