A Paired-Vehicle Recourse Strategy for the Vehicle Routing Problem with Stochastic Demands

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Abstract

This paper presents a paired-vehicle recourse strategy for the vehicle routing problem with stochastic demands (VRPSD). In the VRPSD, a fleet of homogeneous capacitated vehicles is dispatched from a terminal to serve single-period customer demands which are known in distribution when planning but only revealed with certainty upon vehicle arrival. While most existing research for this problem focuses on recourse strategies where each vehicle operates independently, this paper alternatively considers a strategy in which vehicles may be coordinated in pairs. A tabu search heuristic is developed to find good solutions to VRPSD instances with homogeneous customer demand distributions given this alternative recourse strategy. Finally, a computational study on a set of test problems with a variety of demand distributions reveals that the paired recourse strategy may lead to expected travel cost savings of 3% to 25% on problems with 50 customers or greater.

1 Introduction

This paper considers a single-period, single-depot vehicle routing problem in which a fleet of homogeneous capacitated vehicles is dispatched to serve the demands of customers. The set of customers that each vehicle can serve in the period is assumed to be constrained primarily by the maximum load capacity of the vehicle.

When customer load demands are not known with certainty when planning, the routing problem is referred to as the Vehicle Routing Problem with Stochastic Demands (VRPSD). There are many real-world application problems that are best modeled using VRPSD formulations. Examples
include local deposit collection from bank branches, LTL package collection, garbage collection, home heating oil delivery, and forklift routing. Applications are discussed in more detail in Erera (2000) and Bertsimas and Simchi-Levi (1996), and in most of the other references cited in this paper.

Methodologies for planning vehicle routes for systems with stochastic demands can be grouped roughly into two classes: chance-constrained approaches and recourse approaches. As noted in Erera and Daganzo (2003), recourse approaches for such problems have traditionally utilized variants of a simple recourse strategy which we denote the detour-to-depot (DTD) operating scheme. Under the detour-to-depot scheme, vehicles operate independent \textit{a priori} tours of customers, and detour to the depot to unload or reload when capacity is reached before tour completion. Erera and Daganzo (2003) classifies this operating scheme as type $E_1$, where ‘$E$’ indicates that the recourse action is applied as the result of an event (the vehicle reaching capacity) and the ‘1’ indicates that the recourse control scheme replans each vehicle independently.

The DTD scheme is appealing computationally, since under typical demand distribution assumptions the total expected recourse cost expression is separable by tour. Such a scheme may also be appealing in practice, since the recourse control is very simple to implement and requires no centralized replanning. If the approach were used to develop fixed routes to be operated each period, the resulting solution would have the property that each vehicle visits the same set of customers each period and thus travels in familiar territory.

This paper considers an alternative recourse strategy for the VRPSD, denoted the \textit{paired locally-coordinated} (PLC) operating scheme. The PLC scheme is a simple extension of the DTD scheme, and similar vehicle pairing schemes were originally introduced and analyzed via continuous approximation techniques in Erera (2000). Under the PLC scheme considered herein, \textit{a priori} vehicle tours are paired. If a vehicle exceeds capacity during operation, its partner adds any unserved customers to the end of its tour, which it then operates to completion using the detour-to-depot scheme. The PLC scheme is an $E/2$ recourse strategy, where an event initiating a recourse decision may result in replanning for two vehicles simultaneously.

The PLC strategy attempts to utilize risk pooling to reduce system costs. In this paper we show that a simple version of the scheme can be configured with computational effort not much greater than that required for detour-to-depot. Furthermore, the scheme remains practical and requires at most local communication between each pair of vehicles. When used as a generator of fixed routes,
the PLC strategy will yield a solution in which each customer is served by one of two vehicles (the initial vehicle or its partner) each period; thus, most of the advantages of customer-vehicle familiarity are preserved.

In this paper, we develop a heuristic for determining good solutions to VRPSD instances with homogeneous customer demand distributions. Homogeneous demands are assumed primarily for ease of exposition. A computational study indicates that systems configured using the PLC recourse scheme may realize substantial reductions in total expected fleet travel cost.

The primary contributions of this paper include:

• Specification of a simple, practicable, configurable paired locally-coordinated recourse strategy for the VRPSD;

• Development of a computational configuration approach for the PLC scheme for VRPSD instances with homogeneous demands via tabu search; and

• A computational study of the PLC scheme that indicates potential significant benefits in terms of reduced expected travel cost when compared to the DTD scheme.

The remaining sections of this paper are organized as follows. Section 2 defines the stochastic routing problem under consideration, and briefly reviews relevant research. Section 3 then defines the specific PLC recourse scheme employed in this paper. Section 4 develops a tabu search heuristic for generating solutions to the VRPSD using PLC recourse. Finally, Section 5 presents the results of computational study of the heuristic and compares the resulting solutions to those generated using the DTD recourse scheme for a set of problem instances.

2 Problem Definition

The classical Vehicle Routing Problem (VRP) is defined as follows. Consider a complete undirected network \( G = (V, E) \) where \( V = \{v_0, v_1, v_2, \ldots, v_n\} \) is the vertex set and \( E = \{(v_i, v_j) : i \neq j, v_i, v_j \in V\} \) is the edge set. A depot is located at vertex \( v_0 \), which serves as both the base for a fleet of vehicles and also the destination (or source) of goods. Each remaining vertex corresponds to a customer location, from which (or to which) goods are to be collected (or distributed). A travel cost matrix \( D = \{d(v_i, v_j)\} \) is defined on \( E \). Each vehicle has homogeneous load capacity \( Q \), and each customer location \( v_i \) requires that a load of size \( q_i \) is collected (or delivered) during the
operating period. The VRP then is to determine a set of vehicle tours such that the sum of the travel costs of all tours is minimum, where each tour begins and ends at the depot, the demand of each customer is completely served by a single vehicle (no splitting), and no vehicle tour exceeds load capacity.

The *Vehicle Routing Problem with Stochastic Demands* (VRPSD) is an extension of the VRP where the customer load demands are random variables known only in distribution when planning; see Dror et al. (1989) and Gendreau et al. (1996b) for an introduction. Let $\xi_i$ be the random variable for demand at customer location $v_i$. Different variants of the VRPSD consider different assumptions about customer demand distributions and when uncertain quantities are revealed. In this paper, customer demands $\xi_i$ are assumed to be discrete, independent and identically-distributed random variables with finite support $\mathcal{X}$, where $\min\{\mathcal{X}\} \geq 0$ and $\max\{\mathcal{X}\} < Q$. The individual demand for a customer is observed with certainty when a vehicle arrives at the customer location. In the remainder of this paper, we will assume that vehicles are collecting freight from customers.

As noted earlier, solution techniques for VRPSD problems typically utilize either a chance-constrained or recourse approach. Important research examples of the chance-constrained approach for such problems include Stewart and Golden (1983) and Laporte et al. (1989) which propose mathematical programming methods for solution generation. Chance-constrained approaches do not explicitly estimate the expected recovery costs implied by a solution. Addressing this shortcoming, Dror and Trudeau (1986) develops heuristics that use penalty costs to approximate the expected additional recovery cost of a solution.

Full recourse approaches for the VRPSD attempt to determine a set of *a priori* vehicle tours that minimize the total expected cost of operations given a recourse strategy that specifies a recovery action to apply when the *a priori* solution requires replanning. Bertsimas (1992) considers a problem variant where the goal is to build a single *a priori* tour with minimum expected cost; this reference proposes and analyzes the DTD operating scheme, and develops heuristics with asymptotic worst-case bounds including an asymptotically-optimal heuristic. Since under certain distributional assumptions, the expected recourse cost is separable by tour and computable, research has investigated algorithmic approaches based on stochastic integer programming for determining solutions to multiple vehicle problems. Following Laporte and Nobert (1980) and Laporte and Louveaux (1993), Gendreau et al. (1995) proposes an exact algorithm using the L-shaped approach and provides computational results for problems with two vehicles; Laporte et al. (2002) provides
improved lower bounds and optimality cuts, and shows that problems with up to 100 customers and two vehicles can be solved within ten minutes of computation time. Gendreau et al. (1996a) alternatively develops a tabu search heuristic for the problem. Alternative solution approaches such as the sample average approximation approach may provide a method for developing near-optimal solutions to problems with larger vehicle fleets; see Verweij et al. (2003) for computational results on the traveling salesperson problem with random travel times.

3 A Paired Locally-Coordinated Recourse Scheme for VRPSD

The DTD recourse strategy is simple and appealing, but it may be possible to achieve lower expected cost solutions by pooling the capacity of multiple vehicles and thus increasing capacity utilization. The simplest, and perhaps most practical, way of doing so is to coordinate the operations of *pairs* of vehicles. In this research, we investigate the performance of a simple operating scheme that coordinates the operations of vehicle pairs, denoted PLC.

Under the PLC scheme, each customer is assigned in the first stage to a single *a priori* tour, each starting and ending at the depot. Some (but not necessarily all) tours are matched together to create a tour pair, but no tour is included in more than one tour pair. One of the tours in each pair is denoted *a priori* as a *type I* tour, and the other as a *type II* tour; see Figure 1. Each tour that is not paired with another may also be considered a type II tour.

In the second stage, each vehicle begins visiting customers in the order specified by its *a priori* tour. At some point, an individual vehicle may experience a *tour failure* due to lack of capacity. A tour failure is said to occur at a customer if adding its demand to the load already carried by the vehicle would exceed capacity $Q$, or exactly meet capacity $Q$ if the customer is not the final visit on the tour. Since customer demand may not be split, the vehicle will collect the demand at a failure point only if adding it would exactly meet vehicle capacity; otherwise, this demand will be collected after some recourse action. If a vehicle operating a type I tour fails at a customer visit, it returns to the depot leaving a set of unserved customers. These unserved customer are then appended to the end of the *a priori* type II tour for the vehicle’s partner. If a vehicle operating a type II tour fails, it detours to the depot to unload and then resumes collections at the first remaining unserved customer in its tour. Note that each such vehicle must wait at the final customer on its *a priori* tour until the vehicle serving its paired type I tour begins return travel to the depot.
Figure 1: Paired tours operated under the PLC scheme: on the left, the \textit{a priori} tours, and on the right, a possible recourse outcome.

PLC is clearly an asymmetric operating scheme, since the type II vehicle in each pair is designated \textit{a priori}. A symmetric alternative scheme might determine the “type II” tour only after the first vehicle failure occurs; \textit{e.g.}, the first tour to fail would be “type I”, and its remaining unserved customers added to end of the \textit{a priori} partner tour. While such a scheme is appealing practically and may lead to additional operating cost benefit, we do not analyze it in this paper since we believe the additional benefit will be small, and further since analysis of such a strategy is more complex and necessitates consideration of travel and service times, which we ignore here for simplicity.

4 A Tabu Search Heuristic for the VRPSD with PLC Recourse

This section presents a tabu search heuristic for determining \textit{a priori} tours and pairs for routing systems operated with the PLC recourse scheme. Like all tabu search approaches, the method begins with an initial solution, and moves each iteration to the best neighbor of the current solution (regardless of whether the objective function improves). Cycling is avoided by placing the previous solution in a tabu list, making it inaccessible for a number of iterations.

Tabu search has been applied to deterministic combinatorial problems like the VRP by many
researchers. For the VRP with stochastic demands and customers, Gendreau et al. (1996a) proposes
the TABUSTOCH heuristic and shows that it is effective by comparing results to optimal solutions
for small problems. The tabu search method proposed in this section utilizes a neighborhood
structure very similar to TABUSTOCH. Unlike Gendreau et al. (1996a), we assume for simplicity
that customer demands are identically distributed, and therefore the approach that we develop
evaluates the cost of each “move” from a solution to a neighbor exactly rather than approximately.
We note that the approach could be extended to problems with heterogeneous customer demand
distributions without too much difficulty, following the approach in Gendreau et al. (1996a).

In the following subsections, we present the key building blocks of the tabu search heuristic.
We then piece the components together and describe the heuristic.

### 4.1 Failure Probabilities

To evaluate the expected travel cost of an *a priori* tour or pair of tours, we first consider the
probability of a tour failure. Both type I and type II tours may fail during operation. Note that
each vehicle operating a type I tour may fail at most once, while each vehicle operating a type II
tour may fail many times. Recall that under the definition presented earlier, a vehicle is said to fail
at a customer if loading that customer’s demand would exceed or exactly meet vehicle capacity.

Let \( p^\ell \) be the probability that a customer demands exactly \( \ell \) units (note that \( p^\ell = 0 \) for \( \ell \notin \mathcal{X} \)).

Given a tour of customers, let \( \Pi_i(h) \) be the probability that the first \( i \) customers in the tour demand
a cumulative quantity of \( h \). These probabilities can be computed recursively:

\[
\Pi_i(h) = \begin{cases} 
\begin{align*}
 p^h & \quad \text{for } i = 1 \\
 \sum_{\ell=0}^h p^\ell \Pi_{i-1}(h-\ell) & \quad \text{for } i \geq 2
\end{align*}
\end{cases} \tag{1}
\]

Consider now a specific pair of vehicle tours, denoted pair \( k \), where \( t^1_k \) customers have been
assigned to the type I tour and \( t^2_k \) customers to the type II tour. Let \( \bar{q}^k_\eta \) be the probability that the
capacity of the vehicle serving the type I tour in the pair is exceeded and let \( \bar{\bar{q}}^k_\eta \) be the probability
that the capacity is exactly met at the \( \eta \)-th customer on the tour. The following lemma establishes
the probabilities \( \bar{q}^k_\eta \) and \( \bar{\bar{q}}^k_\eta \):

**Lemma 1 (Failure Probabilities for Type I Tour)** *Given a tour pair \( k \) with \( t^1_k \) customers assigned to the type I tour,*
\[ \tilde{q}_\eta^k = \begin{cases} \sum_{\ell>Q} p^\ell = 0 & \text{for } \eta \in \{0, 1\} \\ \sum_{g=1}^Q \left( \sum_{\ell>g} p^\ell \right) \Pi_{\eta-1}(Q-g) & \text{for } 2 \leq \eta \leq t_k^1 \end{cases} \]  

and

\[ \bar{q}_\eta^k = \begin{cases} p^Q = 0 & \text{for } \eta \in \{0, 1\} \\ \sum_{g=1}^Q p^g \Pi_{\eta-1}(Q-g) & \text{for } 2 \leq \eta < t_k^1 \end{cases} \]

Furthermore, the probability that the vehicle completes its tour without failure, which is denoted as \( \bar{q}_0^k \), can be calculated as

\[ \bar{q}_0^k = \begin{cases} 1 & \text{for } t_k^1 = 0 \\ \sum_{h=0}^Q \Pi_{t_k^1}(h) & \text{otherwise} \end{cases} \]

Note of course that \( t_k^1 = 0 \) indicates that no customers are assigned to the type I tour of the pair; in this case, no vehicle is needed. Also note that a failure is not deemed to occur if the vehicle serving the type I tour reaches capacity exactly at its \( t_k^1 \)-th customer.

Now consider vehicles serving type II tours. Since customer demands are homogeneous and not splittable, it is straightforward to develop a recursive procedure for calculating \( \bar{f}_i \) and \( \bar{\bar{f}}_i \), which we use to represent respectively the probabilities that the capacity of a vehicle serving a type II tour is exceeded and exactly met at the \( i \)-th customer it visits. Note that the \( i \)-th customer may be one of its \textit{a priori} customers \( (i \leq t_k^2) \) or one of the customers reassigned from the failed type I vehicle \( (i > t_k^2) \). Note further that the \( \bar{q}_\eta^k \) and \( \bar{\bar{q}}_\eta^k \) terms for \( \eta > 0 \) in Lemma 1 are the same for all tour pairs, and simply represent the probabilities (which we now denote \( \tilde{q}_\eta \) and \( \bar{\tilde{q}}_\eta \)) of any vehicle failing for the first time at its \( \eta \)-th customer. When such a vehicle fails, it returns to the depot, unloads, and returns empty to resume operations at the first remaining unserved customer. Thus, the failure probabilities \( \bar{f}_i \) and \( \bar{\bar{f}}_i \) can be calculated recursively by conditioning on the location of the first vehicle failure, as given by the following lemma:

**Lemma 2 (Failure Probabilities for a Vehicle Serving a Type II Tour)** Given a problem with \( n \) customers, the following failure probabilities for any vehicle serving a type II tour can be computed a priori:
\[
\bar{f}_i = \begin{cases} 
q_i = 0 & \text{for } i \in \{0, 1\} \\
q_i + \sum_{j=1}^i \bar{q}_j \bar{f}_{i-j+1} + \sum_{j=1}^{i-1} \bar{q}_j \bar{f}_{i-j} & \text{for } 2 \leq i \leq n
\end{cases}
\]  

and

\[
\bar{p}_i = \begin{cases} 
\bar{q}_i = 0 & \text{for } i \in \{0, 1\} \\
\bar{q}_i + \sum_{j=1}^i \bar{q}_j \bar{f}_{i-j+1} + \sum_{j=1}^{i-1} \bar{q}_j \bar{f}_{i-j} & \text{for } 2 \leq i \leq n
\end{cases}
\]

4.2 Objective Function

The objective function to be minimized is the total expected travel cost required by all vehicles. Let \(x^\nu\) be the vector of decision variables representing the a priori solution (assignment of customers into ordered tours, and matching of tours into pairs) at iteration \(\nu\) of the heuristic. Since the expected cost of any tour pair can be computed independently from the cost of any other tour pair, first consider some tour pair \(k\). Suppose that the type I tour of tour pair \(k\) is given by \((v_{0}^{1}, v_{1}^{1}, ..., v_{t_{k}^{1}}^{1}, v_{t_{k}^{1}+1}^{1} = v_{0}^{1})\), and that the type II tour is \((v_{0}^{2}, v_{1}^{2}, ..., v_{t_{k}^{2}}^{2}, v_{t_{k}^{2}+1}^{2} = v_{0}^{2})\). Note that both \(v_{0}^{1}\) and \(v_{0}^{2}\) represent the depot, and \(t_{k} = t_{k}^{1} + t_{k}^{2}\) gives the total number of customers served by the tour pair.

To compute \(T^k(x^\nu)\), the expected cost of operating tour pair \(k\), we first develop an expression for \(\bar{T}^k(\nu)\), the expected cost of the tour pair conditional on the case when the capacity of the vehicle serving the type I tour is exceeded at the \(\eta\)-th customer \((\eta \geq 1)\) on its tour, or the vehicle does not fail \((\eta = 0)\). Similarly, we develop an expression for \(\bar{T}^k(\nu)\), the expected cost of the tour pair conditional on the case when the capacity is exactly met at the \(\eta\)-th customer \((\eta \geq 1)\). The results are established in the following proposition.

**Proposition 1** The expected cost of tour pair \(k\) given that the capacity of the vehicle serving the type I tour is exceeded at its \(\eta\)-th customer \((1 \leq \eta \leq t_{k}^{1})\), or it does not fail \((\eta = 0)\) is

\[
\bar{T}^k(\nu) = \sum_{i=0}^{\eta-1} d(v_{i}^{1}, v_{i+1}^{1}) + d(v_{i}^{1}, v_{0}^{1}) + \sum_{i=0}^{t_{k}^{1}-1} d(v_{i}^{1}, v_{i+1}^{2}) + d(v_{i}^{2}, v_{1}^{1}) + \sum_{i=\eta}^{t_{k}^{1}} d(v_{i}^{1}, v_{i+1}^{1})
\]

\[
+ \sum_{i=1}^{t_{k}^{2}} \bar{f}_i (d(v_{i}^{2}, v_{0}^{2}) + d(v_{0}^{2}, v_{i}^{2})) + \sum_{i=1}^{t_{k}^{2}-1} \bar{f}_i (d(v_{i}^{2}, v_{0}^{2}) + d(v_{0}^{2}, v_{i}^{2}) - d(v_{i}^{2}, v_{i+1}^{2}))
\]

\[
+ \bar{f}_{t_{k}^{2}} (d(v_{t_{k}^{2}}^{2}, v_{0}^{2}) + d(v_{0}^{2}, v_{t_{k}^{2}}^{1}) - d(v_{t_{k}^{2}}^{2}, v_{t_{k}^{2}+1}^{1})) + \gamma^k(\eta)
\]
The expected cost of tour pair \( k \) given that the capacity of the vehicle serving the type I tour is exactly met at its \( \eta \)-th customer \((1 \leq \eta < t_k^1)\) is

\[
\overline{T}_\eta^k(x^\nu) = \sum_{i=0}^{\eta-1} d(v^1_i, v^1_{i+1}) + d(v^1_\eta, v^1_0) + \sum_{i=0}^{t_k^2-1} d(v^2_i, v^2_{i+1}) + d(v^2_{t_k^2}, v^1_{\eta+1}) + \sum_{i=\eta+1}^{t_k^1} d(v^1_i, v^1_{i+1})
\]

\[
+ \sum_{i=1}^{t_k^2} \bar{f}_i (d(v^2_i, v^2_0) + d(v^2_0, v^2_{i+1})) + \sum_{i=1}^{t_k^2-1} \bar{f}_i (d(v^2_i, v^2_0) + d(v^2_0, v^2_{i+1}) - d(v^2_i, v^2_{i+1}))
\]

\[
+ \bar{f}_{t_k^2} (d(v^2_{t_k^2}, v^2_0) + d(v^2_0, v^1_{\eta+1}) - d(v^2_{t_k^2}, v^1_{\eta+1})) + \gamma_k(\eta + 1)
\]

(8)

where

\[
\gamma_k(\eta) = 0 \quad \text{for} \quad \eta = 0
\]

(9)

and

\[
\gamma_k(\eta) = \sum_{i=\eta}^{t_k^1} \bar{f}_{t_k^2-\eta+i+1} (d(v^1_i, v^1_0) + d(v^1_0, v^1_i))
\]

\[
+ \sum_{i=\eta}^{t_k^1-1} \bar{f}_{t_k^2-\eta+i+1} (d(v^1_i, v^1_0) + d(v^1_0, v^1_{i+1}) - d(v^1_i, v^1_{i+1})) \quad \text{for} \quad 1 \leq \eta \leq t_k^1
\]

(10)

**Proof.** Consider first expression (7). The first line gives the combined travel cost of both vehicles, assuming that the vehicle serving the type II tour does not fail. The remainder of the expression adjusts this cost to account for the recourse actions of vehicle serving the type II tour. While serving its original customers, note that if this vehicle reaches capacity exactly at its \( t_k^2 \)-th customer, it detours to the depot and immediately proceeds to the \( \eta \)-th customer of the type I tour. The function \( \gamma_k(\eta) \) simply determines the cost of type II vehicle recourse while serving customers originally assigned to the type I tour, given that customer \( v^1_\eta \) is the first unserved customer on the type I vehicle tour.

Expression (8) is similar, where again the first line gives the combined travel cost of the tours assuming that the vehicle serving the type II tour does not fail. Note in this case that customer \( v^1_\eta \) is fully-served before the vehicle serving the type I tour returns to the depot, and therefore the remaining terms are adjusted accordingly. \( \square \)

Using the failure probabilities \( \overline{q}_\eta^k \) and \( \bar{q}_\eta^k \), we can now compute \( T_k(x^\nu) = \sum_{\eta=0}^{t_k^1} \overline{q}_\eta^k \overline{T}_\eta^k(x^\nu) + \sum_{\eta=1}^{t_k^1-1} \overline{q}_\eta^k \bar{T}_\eta^k(x^\nu) \). The result is established in the following proposition, provided without proof.
Proposition 2 The total expected cost of tour pair \( k \) is

\[
T^k(x^{\nu}) = \sum_{i=0}^{t_1^k} d(v_1^i, v_{i+1}^i) + \sum_{i=0}^{t_2^k} d(v_1^i, v_{i+1}^i) + \sum_{i=1}^{t_2^k} f_i (d(v_1^i, v_0^i) + d(v_2^i, v_0^i)) + \sum_{i=1}^{t_2^k-1} \bar{f}_i (d(v_2^i, v_0^i) + d(v_2^{i+1}, v_{i+1}^i) - d(v_1^i, v_{i+1}^i)) + \sum_{\eta=0}^{t_1^k} q^k_\eta\left[ d(v_1^i, v_0^i) + d(v_2^i, v_1^i) - d(v_1^i, v_{\eta+1}^i) + \bar{f}_i^k (d(v_2^i, v_0^i) + d(v_2^i, v_1^i) - d(v_1^i, v_{\eta+1}^i)) + \gamma^k(\eta) \right] + \sum_{\eta=1}^{t_1^k-1} q^k_\eta\left[ d(v_1^i, v_0^i) + d(v_2^i, v_1^i) - d(v_1^i, v_{\eta+1}^i) + \bar{f}_i^k (d(v_2^i, v_0^i) + d(v_2^i, v_1^i) - d(v_1^i, v_{\eta+1}^i)) + \gamma^k(\eta+1) \right]
\]

Note that the homogeneous demand assumption enables efficient computation of \( T^k(x^{\nu}) \), since the probabilities \( \bar{f}, \bar{f}, \bar{q}, \), and \( \bar{q} \) can be computed once initially; determining the cost of a tour using the lemma requires \( O(a^2 + b) \) effort, where \( a = t_1^k \) and \( b = t_2^k \), and is therefore very fast for realistically-sized problems.

The value of the objective function at iteration \( \nu \) is simply the sum of the expected costs of all tour pairs,

\[
T(x^{\nu}) = \sum_{k=1}^{m^{\nu}} T^k(x^{\nu}),
\]

where \( m^{\nu} \) is the number of tour pairs at iteration \( \nu \). Again, recall that any vehicle tour not paired with another can simply be considered a tour pair with no type I tour.

It is also interesting to consider the case where a tour “pair” assigns all customers to the type I tour, leaving the vehicle serving the type II initially at the depot with no \textit{a priori} customers. It turns out that an alternative configuration in which these customers are instead assigned entirely to a type II tour in the same sequence (with no partner type I tour) has the same expected cost, as established in the following proposition:

Proposition 3 A tour pair with a sequence of \textit{a priori} customers assigned only to the type I tour with no customers assigned to the type II tour results in the same expected cost as a tour pair with the customers assigned in the same sequence to a type II tour, with no partner type I tour.

Proof. Consider a tour pair with all customers assigned to the type II tour. In this case, \( t_1^k = 0 \) and \( t_k = t_2^k \). Suppose that the \textit{a priori} tour assigned to the vehicle is \( \{0, 1, 2, ..., t_k, t_k + 1 \equiv 0\} \). From Proposition 2,
All customers are assigned to the Type II vehicle in the first stage

All customers are assigned to the Type I vehicle in the first stage

Figure 2: The expected cost of a tour pair with all customers assigned to the type I tour has the same expected cost as the same tour pair with all customers assigned to a type II tour.

\[
T^k(x^\nu) = \sum_{i=0}^{t_k-1} d_{i,i+1} + \sum_{i=1}^{t_k} \bar{f}_i(d_{i,0} + d_{0,i}) + \sum_{i=1}^{t_k-1} \bar{f}_i(d_{i,0} + d_{0,i+1} - d_{i,i+1}) + q^k_0 d_{t_k,0} \\
= \sum_{i=0}^{t_k} d_{i,i+1} + \sum_{i=1}^{t_k} \bar{f}_i(d_{i,0} + d_{0,i}) + \sum_{i=1}^{t_k-1} \bar{f}_i(d_{i,0} + d_{0,i+1} - d_{i,i+1}) , \tag{13}
\]

since \( q^k_0 = 1 \).

Now suppose that the same sequence of customers were instead all assigned to the type I tour, so that \( t_k^2 = 0 \) and \( t_k = t_k^1 \). In this case, the type II tour has no customers in its \textit{a priori} tour, so the vehicle serving it waits at the depot for the failure of the type I tour. Again substituting directly into the expected cost expression of Proposition 2 yields

\[
T^k(x^\nu) = \sum_{i=0}^{t_k} d_{i,i+1} + \sum_{\eta=1}^{t_k} \bar{q}_\eta [d_{\eta,0} + d_{0,\eta} + \gamma(\eta)] + \sum_{\eta=1}^{t_k-1} \bar{q}_\eta [d_{\eta,0} + d_{\eta,\eta+1} - d_{\eta,\eta+1} + \gamma(\eta + 1)] \tag{14}
\]

where

\[
\gamma(\eta) = \sum_{i=\eta}^{t_k} \bar{f}_{i+1-\eta}(d_{i,0} + d_{0,i}) + \sum_{i=\eta}^{t_k-1} \bar{f}_{i+1-\eta}(d_{i,0} + d_{0,i+1} - d_{i,i+1}) \quad \text{for} \ 1 \leq \eta \leq t_k . \tag{15}
\]
Substituting (15) into (14), and performing a bit of algebra yields:

\[
T^k(x^*) = \sum_{i=0}^{t_k} d_{i,0} + \sum_{i=1}^{t_k} \left( (d_{i,0} + d_{0,i}) \left( \bar{q}_i + \sum_{j=1}^i \bar{q}_j \bar{f}_{i-j-1} + \sum_{j=1}^{i-1} \bar{q}_j \bar{f}_{i-j} \right) \right) \\
+ \sum_{i=1}^{t_k-1} \left( (d_{i,0} + d_{0,i+1} - d_{i,i+1}) \left( \bar{q}_i + \sum_{j=1}^i \bar{q}_j \bar{f}_{i-j-1} + \sum_{j=1}^{i-1} \bar{q}_j \bar{f}_{i-j} \right) \right).
\] (16)

Thus, we can conclude that expression 13 and expression 16 are equal given the definitions of \( \bar{f} \) and \( \bar{\bar{f}} \) in Lemma 2. \( \square \)

Proposition 3 is helpful, since it implies that any tour pair where all of the customers are assigned \textit{a priori} to a single tour (either type I or type II) can be operated with identical expected travel cost using a single vehicle.

4.3 Initial Solution

We find an initial solution to the problem using a sweep method. The method first determines the number of customers to be assigned to each vehicle in a tour pair using an arbitrary service level probability parameter, \( p_\alpha \). Given \( p_\alpha \), we set the maximum number of customers \( \bar{t}_2^k \) to be assigned to an initial \textit{a priori} type II tour to be

\[
\bar{t}_2^k = \max \left\{ i \mid 1 \leq i \leq n, \sum_{h=0}^{Q} \Pi_i(h) \geq p_\alpha \right\}; \tag{17}
\]

\( p_\alpha \) is thus the probability that a vehicle serving a type II tour does not require a recourse detour to the depot \textit{before} covering any of the customers of its type I partner. We also use \( p_\alpha \) to set the maximum number of customers \( \bar{t}_1^k \) to assign \textit{a priori} to any initial type I tour:

\[
\bar{t}_1^k = \max \left\{ i \mid 1 \leq i \leq n, \sum_{h=0}^{2Q} \Pi_i(h) \geq p_\alpha \right\} - \bar{t}_2^k \tag{18}
\]

Note then that the total customer demand initially assigned to a pair of vehicles is such that the combined capacity should be sufficient to serve the customers without a recourse detour with probability \( p_\alpha \), and that more customers will be assigned to each type I tour than to each type II tour.
tour. We note for clarity that these maximums apply only to the initial tours assigned to vehicles, and may be exceeded during the subsequent tabu search.

The simple sweep initialization method starts with a ray directed eastward from the depot, and rotated counter-clockwise. Tour pairs are constructed by assigning the first $t_k^1$ customers encountered by the ray to the type I tour of the first pair, and the next $t_k^2$ customers to the type II tour of the first pair, and continues until all vertices are assigned to some tour of some tour pair. Each type I tour is operated in a counter-clockwise direction, while each type II tour is operated in a clockwise direction.

During the development of our heuristic, we experimented with a number of possible values for $p_\alpha$, and found best final solutions to test problems when $p_\alpha = 0.95$. This selection results in conservative initial solutions where tour pairs require a recourse detour with probability 0.05. We also experimented with different methods for generating initial solutions. In general, different approaches led to very minor differences in final solutions, and the sweep approach described in this section delivered the best results on average among the approaches considered.

### 4.4 Neighborhood Structure

The tabu search heuristic moves during each iteration from the current solution to a neighbor. We define the $(p, r, q)$ neighborhood $N(p, r, q, x)$ of a solution $x$ as all solutions that are generated by removing a customer from its tour and reinserting that customer in a different location, where each of $q$ randomly selected customers is selected for removal and is reinserted either immediately before or immediately after one of $p$ randomly selected customer neighbors from its set of $r$ nearest neighbors ($p \leq r$). Additionally, we augment the neighborhood to allow the creation of new partner tours for unpaired vehicles, and to also potentially generate new tour pairs. The latter option is considered each iteration, while a new partner tour is generated if the selected neighbor is in an unpaired tour. Note that moves that result in the deletion of a tour or tour pair are also allowed.

This neighborhood structure is similar to the one described in Gendreau et al. (1996a) where the $p$ nearest neighbors are considered for each of $q$ randomly selected customers, defining a $\tilde{N}(p, q, x)$ neighborhood. Thus, the same customer neighbors are considered for a randomly selected customer at every iteration when using the $\tilde{N}(p, q, x)$ neighborhood. Our computational experience indicated that using the more randomized $N(p, r, q, x)$ neighborhood produced slightly better solutions for systems operating under both the DTD and PLC recourse strategies for the vast majority of test
problems.

4.5 Evaluating Cost of Moves

Since the probabilities $\bar{q}_i$, $\bar{q}_i$, $\bar{f}_i$ and $\bar{\bar{f}}_i$ can be computed once during initialization for each $i \leq n$, it is not prohibitive to compute the exact expected cost of the new tour pairs generated by any customer move. At the beginning of each iteration $\nu$, we have stored the expected costs of all of the tour pairs in the current solution. Let $F(i, j)$ be the change in solution cost generated by removing customer $v_i$ from its tour and placing it in the tour pair with neighbor $v_j$. Let $k^\nu(\ell)$ denote the tour pair index of $v_\ell$ at the beginning of iteration $\nu$, and $T^k(x^\nu)$ denote the current value of the expected cost of tour pair $k$. Similarly, let $T^k(x^{\nu+1})$ denote the resulting value of the expected cost of tour pair $k$, if the considered move were implemented. Assume $k^\nu(0) = 0$ and $T^0(x^\nu) = 0$ where $v_j = v_0$ represents moving $v_i$ to a new empty tour pair. Then,

$$F(i, j) = \begin{cases} T^{k^\nu+1(i)}(x^{\nu+1}) - T^{k^\nu(i)}(x^\nu) & \text{if } k^\nu(i) = k^\nu(j) \\ T^{k^\nu+1(i)}(x^{\nu+1}) + T^{k^\nu(i)}(x^{\nu+1}) - T^{k^\nu(i)}(x^\nu) - T^{k^\nu(j)}(x^\nu) & \text{if } k^\nu(i) \neq k^\nu(j) \end{cases} \quad (19)$$

4.6 Tabu Moves

If a customer is moved outside of its current tour during iteration $\nu$, the reverse of this move is made tabu until iteration $\nu + \theta$, where $\theta$ is randomly selected in the interval $[n - 5, n]$. Random tabu durations are used to further reduce the likelihood of solution cycling.

Additionally, since customers can be moved within their current tour, an additional set of tabu moves is defined to prevent immediate cycling. The move of $v_j$ before (or after) $v_i$ is made tabu if these two vertices belong to the same tour, $v_i$ is the successor (predecessor) of $v_j$, and $v_i$ was moved before (after) $v_j$ in the previous iteration.

4.7 Choosing Next Solution

In general, the heuristic will move from the current solution $x^\nu$ to the best non-tabu solution $x^{\nu+1}$ in $N(p, r, q, x^\nu)$. However, the heuristic can move to a tabu solution $x$ if that solution has the best found cost overall.
4.8 Steps of the Tabu Search Algorithm

The steps of the tabu search algorithm are now summarized in full:

**STEP 1: Initialization**

Set $p := \min\{n - 1, 5\}$ and determine the $r$ closest neighbors of each vertex, where $r = \min\{n - 1, 2p\}$. Construct the initial solution $x$ using the sweep method with $p_a = 0.95$. Set $T^* := T(x)$ and $x^* = x$. Set $q := \min\{n, \mu\}$ where $\mu$ is the number of tours in the initial solution. Initialize counters $t_0 := 0$, $t_1 := 0$, and $t_2 := 100n$, where $t_0$, $t_1$, and $t_2$ are, respectively, the iteration count, the current number of iterations with no improvement in $T^*$, and the maximum number of iterations allowed with no improvement in $T^*$ before termination. Finally, set $s := 1$ where $s$ indicates current run status.

**STEP 2: Neighborhood Search**

Set $t_0 := t_0 + 1$. Consider all candidate moves in $N(p, r, q, x)$, and construct a list $L$ by ranking the moves in nondecreasing order of $F$. Select the first move $(i, j)$ in $L$, and if the move is not tabu or if $T(x) + F(i, j) < T^*$, then let $z$ be the solution generated by making the move. If the move is tabu or does not improve the best solution, consider the next move in $L$ and repeat the check.

**STEP 3: Solution Update**

Set $x := z$ and $T(x) = T(z)$. If $T(z) < T^*$, set $x^* = z$, $T^* = T(z)$ and $t_1 := 0$. Otherwise, set $t_1 := t_1 + 1$. If $t_1 < t_2$, go to Step 2. Else, go to Step 4.

**STEP 4: Intensification, Combination or Termination**

If $s = 1$ and $t_2 = 100n$, set $p := \min\{n - 1, 10\}$, $q := n$, $t_1 := 0$, $t_2 := 20n$, $x := x^*$, $s := 2$ and go to Step 2.

Else If $s = 2$ and $t_2 = 20n$, construct a new solution $y$ by combining unpaired tours (if any) in $x^*$ two by two to form new tour pairs. To create combinations, sort the unpaired tours of $x^*$ according to tour pair index number, and construct a list. Select the first two tours and assign them to the type I and type II tours of a new tour pair respectively. Continue until the list has less than 2 unpaired tours remaining. Set $t_1 := 0$, $t_2 := 10n$, $x := y$, $s := 3$ and go to Step 2.
Else If $s = 3$ and $t_2 = 10n$, stop. The best known solution is $x^*$.

4.9 Additional Remarks

As with all tabu search algorithms, the setting of parameters is an important issue. For $p$ and $q$, the same values as those in TABUSTOCH proposed in Gendreau et al. (1996a) are used. The value of $t_2$ is set larger since the algorithm is expected to run faster because of the homogeneous demand distribution assumption.

5 Computational Study

A computational study on a set of test problems was conducted to assess the quality of systems configured with the PLC recourse strategy, and to verify that the heuristic can generate solutions in reasonable computation times. It is important to note that the results which follow are all generated for VRPSD instances with homogeneous customer demand distributions. While we believe that the results to follow should be fairly representative of a range of potential instances (including those with heterogeneous demand distributions), it is certainly true that the benefit of the PLC strategy will be instance-dependent.

To assess quality, we compare the solutions generated using the PLC recourse strategy to solutions generated using the traditional DTD strategy. We use the tabu search heuristic described above to find solutions to problems using DTD by making the following modifications. First, in the initial solution, only individual tours are constructed by assigning $t_k^2$ number of customers (computed using Expression 17) to each vehicle using the sweep method. The unpaired tours constructed via this approach are all defined to be of type II. Furthermore, the tabu search iterations in this case prevent the generation of any type I tours. The resulting heuristic thus allows the movement of a customer to a different tour, or within its own tour, or to a newly-created type II tour. The tour combination phase is also disabled. To minimize the differences between the heuristics, both use the same values of parameters $r$, $p$, $q$, and $t_2$.

Four different homogeneous discrete demand distributions on the domain $[0, 9]$ are used for the instances; see Table 1. The first two distributions have mean 4.5 units, but the second has smaller variance. The last two distributions have the same variance, but the expected demand of the fourth is larger. The capacity of the vehicle is assumed to be 25 units for all problems.
<table>
<thead>
<tr>
<th></th>
<th>DD 1</th>
<th>DD 2</th>
<th>DD 3</th>
<th>DD 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prob((\xi = 0))</td>
<td>0.100</td>
<td>0.025</td>
<td>0.020</td>
<td>0.020</td>
</tr>
<tr>
<td>Prob((\xi = 1))</td>
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<td>0.050</td>
<td>0.160</td>
<td>0.040</td>
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<tr>
<td>Prob((\xi = 2))</td>
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<td>0.075</td>
<td>0.320</td>
<td>0.050</td>
</tr>
<tr>
<td>Prob((\xi = 3))</td>
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<td>0.150</td>
<td>0.170</td>
<td>0.060</td>
</tr>
<tr>
<td>Prob((\xi = 4))</td>
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<td>0.200</td>
<td>0.090</td>
<td>0.070</td>
</tr>
<tr>
<td>Prob((\xi = 5))</td>
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<td>0.200</td>
<td>0.070</td>
<td>0.090</td>
</tr>
<tr>
<td>Prob((\xi = 6))</td>
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<td>0.150</td>
<td>0.060</td>
<td>0.170</td>
</tr>
<tr>
<td>Prob((\xi = 7))</td>
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<td>0.075</td>
<td>0.050</td>
<td>0.320</td>
</tr>
<tr>
<td>Prob((\xi = 8))</td>
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<td>0.050</td>
<td>0.040</td>
<td>0.160</td>
</tr>
<tr>
<td>Prob((\xi = 9))</td>
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<td>0.025</td>
<td>0.020</td>
<td>0.020</td>
</tr>
<tr>
<td>E[(\xi)]</td>
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<td>4.500</td>
<td>3.230</td>
<td>5.770</td>
</tr>
<tr>
<td>(\sigma_\xi)</td>
<td>2.872</td>
<td>1.987</td>
<td>2.125</td>
<td>2.125</td>
</tr>
</tbody>
</table>

Table 1: The four demand distributions used in the computational study.

5.1 Uniformly-scattered customers in \([0, 100]^2\)

The first set of results uses sets of uniformly-scattered customers. For each instance, \(n\) customers \(\{v_1, ..., v_n\}\) were generated in \([0, 100]^2\) according to a uniform distribution, and \(d(v_i, v_j)\) was defined as the Euclidean distance between \(v_i\) and \(v_j\). Parameter \(n\) was varied between 10 and 150, and ten different instances with different customer locations were generated for each value of \(n\).

Each customer set was solved for every demand distribution, and for two different depot locations. Since the heuristics use a randomized neighborhood, we run each problem instance 5 times using different seeds, and select the solution produced with the smallest objective function value.

We also consider two potential depot locations: a centered location, \(v_0 = (50, 50)\), and a location in the lower left corner of the region, \(v_0 = (0, 0)\). Moving the depot location from \((50, 50)\) to \((0, 0)\) increases the average customer-to-depot distance. Thus, the average cost of a vehicle failure is larger when the depot is located at \((0, 0)\).

Table 2 presents the average percent expected travel cost improvement for the PLC strategy versus the DTD strategy for each value of \(n\), where the average is over the best solutions found for the ten instances at that problem size. The results in the table clearly indicate that systems configured with the PLC strategy can be operated with significantly lower expected costs in most scenarios. The left half of the table presents the improvements when the depot is located in the center of the region \((v_0 = (50, 50))\), and the right half displays the improvements when the depot is located in the lower left corner \((v_0 = (0, 0))\). As expected, the relative benefit of the PLC strategy
<table>
<thead>
<tr>
<th>Number of Customers</th>
<th>Center Depot</th>
<th>Corner Depot</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DD 1</td>
<td>DD 2</td>
</tr>
<tr>
<td>10</td>
<td>0.38%</td>
<td>0.31%</td>
</tr>
<tr>
<td>15</td>
<td>0.89%</td>
<td>0.76%</td>
</tr>
<tr>
<td>20</td>
<td>1.19%</td>
<td>0.83%</td>
</tr>
<tr>
<td>25</td>
<td>2.04%</td>
<td>1.75%</td>
</tr>
<tr>
<td>30</td>
<td>2.35%</td>
<td>2.21%</td>
</tr>
<tr>
<td>35</td>
<td>3.21%</td>
<td>2.92%</td>
</tr>
<tr>
<td>40</td>
<td>2.72%</td>
<td>2.94%</td>
</tr>
<tr>
<td>45</td>
<td>3.70%</td>
<td>2.84%</td>
</tr>
<tr>
<td>50</td>
<td>5.67%</td>
<td>3.88%</td>
</tr>
<tr>
<td>55</td>
<td>4.99%</td>
<td>4.79%</td>
</tr>
<tr>
<td>60</td>
<td>4.88%</td>
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<td>6.71%</td>
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<td>90</td>
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<td>130</td>
<td>11.35%</td>
<td>8.12%</td>
</tr>
<tr>
<td>140</td>
<td>11.85%</td>
<td>8.86%</td>
</tr>
<tr>
<td>150</td>
<td>11.17%</td>
<td>8.47%</td>
</tr>
</tbody>
</table>

Table 2: Average percent improvement in expected travel cost generated by the PLC recourse strategy for test problems; average over ten instances.

The results in Table 2 also indicate that systems with more customers can realize larger relative expected travel cost savings from the PLC recourse strategy. Figure 3 depicts these relationships graphically. In the case of a centered depot, the relative improvement grows approximately linearly with the number of customers, although there is some evidence of the improvement levelling off for the largest problems. With the depot in a corner, however, there is much stronger evidence that the relative benefit of the PLC strategy appears to approach some limiting value as the problem size grows. This pattern can be seen for each of the demand distributions. Such a result is consistent with intuition, since the “penalty” for using the PLC strategy should decrease as the separation...
Figure 3: Graphical depiction of average percent improvement in expected travel cost generated by the PLC recourse strategy for test problems; average over ten instances

between the clusters of customers on the type I and type II tours decreases for each pair, while the “benefit” of such a strategy is limited by the pooled capacity of two vehicles.

It is also important to observe that the relative expected cost savings of the PLC strategy depend on the demand distribution. The discrete uniform demand distribution (number 1) generally appears to lead to the greatest savings, especially for problems with higher average customer-to-depot distance. Comparing the results for distributions 1 and 2, it appears that distributions with higher variability result in larger relative improvements. This result is also consistent with intuition, given that the benefit of the PLC strategy is essentially one of risk pooling. Comparing the results for distributions 3 and 4 which have the same variance but different expected values, it appears that distributions with higher expected value may also generate more savings than those with lower expected value. This relative benefit is larger for problems with a medium number of customers, and is also larger in the case with a centered depot.

In stochastic vehicle routing problems without additional time constraints, solutions typically include multiple a priori tours (or in the case of PLC, multiple tour pairs). While it can be natural to imagine that each a priori tour is operated by a separate vehicle in the fleet, this is clearly not necessary in the absence of time constraints. In a system that uses the detour-to-depot strategy in the absence of time constraints, the return trips to the depot that define the “completion” of an a priori tour can be considered pre-planned detours for a single vehicle that is operating all a
Table 3: Number of planned *a priori* tours and expected detours under DTD and PLC strategies for center depot problems; average over ten instances

*priori* tours in some arbitrary sequence. In order to develop a better understanding of how the PLC strategy balances pre-planned tours with expected detours to achieve travel cost savings, we present in Table 3 and Table 4 a summary of average values for a subset of the instances. For each demand distribution and instance size, we report the average number of *a priori* tours and the average number of expected detours. The total columns represent the sums of these averages, and the improvement columns give the average percentage reduction in these totals from using the PLC strategy.

Note that the percentage reduction in total (pre-planned plus expected) tours ranges from about 5% to 18%. In almost all cases, the PLC strategy reduces the number of *a priori* tours, sometimes by a significant margin. However, the expected total number of detours required is more often higher under the PLC strategy; thus, the average number of detours performed per type II (detouring) vehicle is certainly higher under PLC as well. Again, this paper does not explicitly consider time constraints that would allow determination of an appropriate (or near-optimal) vehicle fleet size to operate either the DTD or PLC strategy feasibly for a given problem instance. We believe this to be an important area for future research.
Table 4: Number of planned a priori tours and expected detours under DTD and PLC strategies for corner depot problems; average over ten instances

<table>
<thead>
<tr>
<th>Number of Customers</th>
<th>Strategy</th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th>Strategy</th>
<th>DD 2</th>
<th></th>
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<th></th>
<th></th>
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<tbody>
<tr>
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<td>1.300</td>
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<td>1.000</td>
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<td></td>
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<tr>
<td>80</td>
<td>DTD</td>
<td>27.200</td>
<td>19.500</td>
<td>21.300</td>
<td>19.2%</td>
<td></td>
<td></td>
<td>PLC</td>
<td>26.500</td>
<td>18.500</td>
<td>16.900</td>
<td>18.0%</td>
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<tr>
<td>90</td>
<td>DTD</td>
<td>32.800</td>
<td>22.600</td>
<td>25.400</td>
<td>21.2%</td>
<td></td>
<td></td>
<td>PLC</td>
<td>32.100</td>
<td>22.100</td>
<td>19.900</td>
<td>21.0%</td>
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<tr>
<td>100</td>
<td>DTD</td>
<td>38.000</td>
<td>27.100</td>
<td>29.900</td>
<td>23.2%</td>
<td></td>
<td></td>
<td>PLC</td>
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<tr>
<td>110</td>
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<td></td>
<td>PLC</td>
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<td>25.8%</td>
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<tr>
<td>120</td>
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<td>38.900</td>
<td>27.9%</td>
<td></td>
<td></td>
<td>PLC</td>
<td>48.900</td>
<td>34.800</td>
<td>30.600</td>
<td>28.4%</td>
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<tr>
<td>130</td>
<td>DTD</td>
<td>54.600</td>
<td>40.400</td>
<td>42.900</td>
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<td></td>
<td></td>
<td>PLC</td>
<td>54.300</td>
<td>39.400</td>
<td>31.500</td>
<td>30.0%</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>140</td>
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<td>45.900</td>
<td>48.800</td>
<td>32.4%</td>
<td></td>
<td></td>
<td>PLC</td>
<td>60.200</td>
<td>44.700</td>
<td>35.600</td>
<td>32.1%</td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>150</td>
<td>DTD</td>
<td>66.600</td>
<td>51.700</td>
<td>54.000</td>
<td>34.6%</td>
<td></td>
<td></td>
<td>PLC</td>
<td>66.300</td>
<td>50.900</td>
<td>37.700</td>
<td>34.1%</td>
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</table>

Finally, it is important to report that the tabu search heuristic generates solutions to these test instances in a reasonable amount of computing time. Table 5 summarizes the minimum, maximum, and average CPU times required by a single run of the heuristic; recall that best solutions are found by taking the best solution of five separate runs for each instance with a different random seed. All computations were performed on an Intel 2.4GHz Pentium processor running Linux with 2GB of memory. Note that problems with 100 customers are generally solved under the PLC recourse strategy in less than five minutes of computing time, and that the largest problems may require 20 minutes. Solution times are generally robust, with the longest run never requiring more than three times the time of the average run for a given problem size. While the PLC strategy generally requires more computing time, the average for the PLC strategy is generally no more than double that for the DTD strategy.
### Table 5: Average run time for tabu search heuristic per instance for PLC and DTD strategies (sec.)

<table>
<thead>
<tr>
<th>Number of Customers</th>
<th>DTD</th>
<th>PLC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Center Depot</td>
<td>Corner Depot</td>
</tr>
<tr>
<td></td>
<td>Min</td>
<td>Max</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>25</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>50</td>
<td>14</td>
<td>45</td>
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<tr>
<td>75</td>
<td>38</td>
<td>134</td>
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<tr>
<td>100</td>
<td>78</td>
<td>241</td>
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<td>110</td>
<td>97</td>
<td>344</td>
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<td>120</td>
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<td>432</td>
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<td>130</td>
<td>154</td>
<td>442</td>
</tr>
<tr>
<td>140</td>
<td>175</td>
<td>497</td>
</tr>
<tr>
<td>150</td>
<td>218</td>
<td>646</td>
</tr>
</tbody>
</table>

5.2 Real-world customer locations

Many real-world routing problem instances have clustered customer locations. Therefore, we also analyze the performance of the PLC strategy using a real-world customer location data set. Figure 4 maps the locations of grocery stores in the city of Istanbul, Turkey, and the distribution center location of a distributor serving those stores. Stores are scattered on both sides of the Bosphorus waterway, which divides the city into two halves which we denote the European (west) side and the Anatolian (east) side. There are 236 and 209 grocery stores receiving service every day on European and Anatolian sides respectively. Since the connection between the two sides is provided by two bridges which experience serious traffic congestion during the day, the distributor uses a separate fleet to serve each side of the city. For our experiment, we therefore consider the European side and the Anatolian side to be two separate geographical location instances, both served by the same depot. It is evident from the figure that these instances contain both clusters of customers in densely populated zones as well as some scattered outlying customers.

Consistent with the previous section, we analyze the performance of both the DTD and PLC operating strategies using the same four homogeneous demand distributions for these two geographic customer instances. For each geographic instance, demand distribution and operating strategy, we find the best expected travel cost solution by running the tabu search heuristic ten times with different random number seeds. Tables 6 and 7 summarize the results.
Figure 4: Locations of grocery stores and grocery distribution center (depot) in Istanbul, Turkey

<table>
<thead>
<tr>
<th></th>
<th>Travel Cost</th>
<th>Total Expected Tours</th>
<th>DTD</th>
<th>PLC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Diff. Imp.( %)</td>
<td>Diff. Imp.( %)</td>
<td>Planned Tours</td>
<td>Detours</td>
</tr>
<tr>
<td>DD1</td>
<td>5516.760 24.34%</td>
<td>18.208 28.17%</td>
<td>62.000</td>
<td>2.647</td>
</tr>
<tr>
<td>DD2</td>
<td>2475.758 12.37%</td>
<td>6.463 12.03%</td>
<td>52.000</td>
<td>1.740</td>
</tr>
<tr>
<td>DD3</td>
<td>2818.082 17.86%</td>
<td>9.533 22.03%</td>
<td>41.000</td>
<td>2.263</td>
</tr>
<tr>
<td>DD4</td>
<td>3040.403 12.15%</td>
<td>8.861 12.67%</td>
<td>68.000</td>
<td>1.910</td>
</tr>
</tbody>
</table>

Table 6: Computational test results for Anatolian store locations

<table>
<thead>
<tr>
<th></th>
<th>Travel Cost</th>
<th>Total Expected Tours</th>
<th>DTD</th>
<th>PLC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Diff. Imp.( %)</td>
<td>Diff. Imp.( %)</td>
<td>Tours</td>
<td>Detours</td>
</tr>
<tr>
<td>DD1</td>
<td>12562.777 26.55%</td>
<td>21.951 28.98%</td>
<td>74.000</td>
<td>1.737</td>
</tr>
<tr>
<td>DD2</td>
<td>5252.642 13.28%</td>
<td>8.002 13.14%</td>
<td>59.000</td>
<td>1.916</td>
</tr>
<tr>
<td>DD3</td>
<td>6196.541 19.62%</td>
<td>10.775 22.05%</td>
<td>47.000</td>
<td>1.877</td>
</tr>
<tr>
<td>DD4</td>
<td>6260.922 12.25%</td>
<td>9.144 11.58%</td>
<td>78.000</td>
<td>0.963</td>
</tr>
</tbody>
</table>

Table 7: Computational test results for European store locations
The results in the tables demonstrate that for problems with clustered customer locations, the PLC operating strategy may lead to significant expected travel cost savings when compared to the DTD operating strategy. Again, the uniform demand distribution leads to the largest percentage cost savings, which for both instances is now about 25%. It is interesting to note that under distribution 3, where customers have high likelihoods of smaller demands and lower likelihoods of larger demands, realizes the second largest expected cost savings under PLC in these cases. This result suggests a beneficial interaction with clustered customers that was not present in the uniform customer location cases of the previous section. Finally, note again that the expected cost benefit of PLC comes along with a significant decrease in the number of pre-planned \textit{a priori} tours and a significant increase in the expected number of total detours.

5.3 Characteristics of solutions with PLC recourse

Since the locally-coordinated strategy PLC presented in this paper is different from those traditionally analyzed, it is useful to provide some discussion regarding characteristics of the best solutions found using the scheme. The generated tour pairs in best solutions often had similar structure across different problems.

First, observations from large numbers of best solutions indicate that tour pairs are usually formed when they are “close” to each other, where closeness can be measured for example by the distance separating the centroids of each tour customer cluster. Such structure is intuitive. In fact, we can refine this statement by instead stating that “adjacent” tours tend to be paired. Adjacency has no precise definition, but one way to think about adjacency is that few if any customers from another tour should lie within the convex hull of the combined set of customers for two adjacent tours.

A perhaps more interesting observation can be made regarding the direction of travel of the two vehicles serving a tour pair. It is important to remember that although the travel costs between locations are symmetric ($d_{ij} = d_{ji}$), the expected travel cost of a tour is not generally symmetric with respect to the travel direction of the vehicle in routing problems with stochastic demands; note that this is true even when customer demand distributions are homogeneous. Thus, a vehicle traversing a tour in the clockwise direction will not incur the same expected cost of a vehicle traversing the tour in the counter-clockwise direction. Under the PLC recourse strategy, travel direction has another effect since it determines how close a vehicle serving a type II tour may
approach unserved customers of the partner type I tour. In the best solutions generated by our heuristic, we observe that for almost all pairs of laterally-adjacent tours, partner vehicles operate in opposite travel directions. For tours that are adjacent radially, we observe that partner vehicles generally operate in the same direction; see the examples in Figure 5.

![Figure 5: Direction of vehicle travel for paired tours; laterally-adjacent (left) and radially-adjacent (right)](image)

It is also interesting to consider the number of customers assigned to each type of vehicle. Since the benefit of the PLC strategy arises when the vehicle serving the type II tour takes on the unserved customers of the type I tour, it is natural to expect that the number of customers assigned to the type I tour should be large enough to ensure that the tour has a high probability of failure. Furthermore, since it is best in general to have few overall failures (especially if customers are farther from the depot), a smaller set of customers is likely to be assigned to the type II tour such that if a failure occurs, it is likely to occur when that vehicle is serving overflow customers from its partner type I tour.

Most realistic instances with many customers will naturally contain a mix of customers that are located further from the depot and those that are located nearer to the depot. It is interesting to investigate how solutions under the PLC strategy allocate customers with different radial distances to the depot to tours. While solutions often contain many laterally-adjacent tour pairs filled with customers with similar distances to the depot, it is sometimes the case that tour pairs will instead be populated with both “nearer” and “further” customers; this is especially true in the corner depot instances. Type II tours usually include further customers than their type I partners, especially near the end of their tours, to minimize the expected cost of radial backtracking. Similarly, nearer
customers are more likely to be found toward the end of the *a priori* type I tours. Such customers are also usually located quite logically to be served with minimum side-tracking by type II vehicles en route to the depot from their final customers.

![Tour pair structures observed in best PLC solutions](image)

Figure 6: Tour pair structures observed in best PLC solutions

Figure 6 illustrates these ideas, presenting the most common tour pair classes that we observe in the best PLC solutions for our computational test problems. Again, most tour pairs are laterally-adjacent pairs such as those depicted in part A of the figure. In addition, we also find a smaller but significant number of tour pairs like those depicted in part B, especially in the corner depot instances. Note that in these typical pairs, the type II tour is assigned fewer customers than its type I partner. Note further that in both of these patterns, the *a priori* type II tour is operated such that the vehicle moves toward the later customers of its partner, and that these customers lie nearby the arc connecting the last customer of the type II tour and the depot.

Part C of the figure depicts another less commonly observed tour pair structure. Here, the type II tour is a bit different from those in parts A and B since it does not visit the most distant
customers in the set covered by the pair. Note however that the final customer in the type II tour is very near to the last customers on the type I tour, and that many of these customers are closer to the depot than the type II tour customers. Lastly, part D depicts a rarely observed, but interesting, tour pair structure. In this case, a type I tour is formed with a set of customers nearby the depot and is paired with a type II tour serving a small number of distant customers where again, the final customers of the type I tour are nearby to the arc connecting the last customer of the type II tour to the depot.

Finally, Figure 7 presents the best solution found under the PLC operating strategy for a 30-customer problem for both center and corner depot case. The reader can verify that the tour pairs exhibit some of the properties discussed above.

Figure 7: Tour pairs in the best solution found for a 30-customer test instance

6 Conclusions

This paper demonstrates that the PLC recourse strategy is a potentially effective operating scheme for vehicle routing systems with uncertain customer demands. Systems operating with the PLC scheme can be configured effectively with tabu search heuristics, with computational effort that is
not much greater than that required for configuring systems that use the standard DTD recourse strategy. Finally, using the strategy may result in significant savings in expected travel costs; computational results for the VRPSD with homogeneous customer demands demonstrate savings of 3% to 25% on test problems with 50 customers and greater.

It would be interesting to extend the results in this paper to treat problems with heterogeneous customer demand distributions. In such problems, the expected cost of a given tour (or tour pair) depends not only on the number of customers included, but also on the relative position within a tour of customers with different demand distributions. An intuitive conjecture might be that for otherwise identical problem instances (same customer locations, and same expected value and variance for the summation of all customer demands) that heterogeneous demand problems will require on average lower expected costs than homogeneous demand problems regardless of the recourse strategy. Whether or not heterogeneous problems would lead to an increase or decrease in the benefit of the PLC strategy over the DTD strategy is not clear.

Finally, future work on stochastic vehicle routing problems needs to consider explicit time constraints on vehicle operations. Both vehicle duration constraints and customer time window restrictions will clearly impact the feasibility of a recourse strategy and therefore the expected travel cost of systems operated under such a strategy. Additionally, considering time constraints will importantly allow researchers to evaluate the fleet size requirements of systems operated under different operating strategies.

References


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