

# An Inventory Control Model with Possible Border Disruptions

Brian M. Lewis, Alan L. Erera, Chelsea C. White III  
School of Industrial and Systems Engineering  
Georgia Institute of Technology, Atlanta, Georgia 30332-0205

## **Abstract**

We consider an infinite-horizon, periodic-review inventory model in which the leadtime probability distribution is dependent on the state of a completely observed, exogenous Markov chain. Order costs are linear, and unsatisfied demand is fully backlogged. It is known that there exists a state-dependent, stationary basestock policy that is optimal for the long-run average cost case. We present a procedure for determining the long-run average cost for a given state-dependent basestock policy and provide sufficient conditions for a state-invariant basestock policy to be optimal, with respect to all state-dependent basestock policies. We then apply these results to analyze the effect of a possible major supply chain disruption (e.g., a border closure) on a firm's inventory and long-run average cost. For this model, we show that a state-invariant basestock policy is optimal and present numerical results that show the optimal basestock level and long-run average cost are more sensitive to the expected duration of a border closure than to the likelihood of a closure. This result has important implications for the cooperation between business and government in disruption management and contingency planning.

# 1 Introduction

Modern global freight transportation and supply chain systems are highly vulnerable to disruptions due to their design and to the volume and value of goods moved. These systems connect an increasingly complex network of products, companies, and nations. According to a 2004 World Trade Organization report, the value of export merchandise transported globally in 2003 was an astonishing \$7.3 trillion. Disruptions to these supply chains not only result in increased operational and recovery costs, but also may adversely affect share holder value. The results of an empirical study in Hendricks and Singhal (2003) show that the mean decrease in firm market value is 10.28% over the two-day period after the public announcement of a supply chain disruption.

The terrorist attacks in the United States on September 11, 2001 provide a specific example of the dramatic impacts of disruptions. According to Bonner (2004), border delays at the US-Canadian border quickly increased from a few minutes to 12 hours, and as a result Ford Motor Company was forced to intermittently idle production at some of its assembly plants. The idling of plants also impacted upstream suppliers by increasing their inventory levels. In the event of another security disruption, border closures are a feasible response that would severely impact international supply chains. A 2003 report from Booz Allen Hamilton (see Gerencser et al. (2003)) presented the results of a port security wargame in which a terrorist attack using “dirty bombs” in intermodal containers was simulated. The actions taken by the participating business and government leaders had significant consequences: every port in the United States was shut down for eight days, requiring 92 days to reduce the resulting backlog of container deliveries, and the total loss to the US economy was \$58 billion, including the costs of spoilage, lost sales/contracts, and manufacturing slowdowns/production halts.

In the new era of supply chain security, Sheffi (2001) states that two challenges face firms: operating efficiently in an environment with heightened security measures designed to prevent disruptions, and planning systems that function efficiently given possible occurrence of disruptions. This paper focuses

on the latter. Since the late 1980s, widespread adoption of just-in-time (JIT) management principles has resulted in safety stock inventory reductions and the use of tight delivery time windows. While JIT principles lead to cost reductions under normal operations, they introduce operational fragility that may increase costs substantially when operations are disrupted. Despite the risk of disruptions, lean management principles need not, and should not, be arbitrarily abandoned for reactionary or just-in-case management approaches. Rather, supply chain disruptions should be incorporated into planning models to develop an appropriate balance between risk buffers (such as safety stock inventory) and lean operations.

We can classify supply system disruptions into two categories: disruptions of product availability at the supplier itself, and disruptions in the transportation of product from supplier to customer. Supplier availability models generally assume that a supplier is either available to supply product or not. These models also either assume zero leadtime, or that at most a single order can be outstanding at any given time. For research in this area, see among others Parlar and Berkin (1991), Weiss and Rosenthal (1992), Parlar and Perry (1995), Parlar et al. (1995), Moinzadeh and Aggarwal (1997), Parlar (1997), Arreola-Risa and DeCroix (1998), and Özekici and Parlar (1999).

There has been little work in the inventory control literature on disruptions in the transportation of product from supplier to manufacturer. Minor disruptions that may cause delays are generally incorporated into the probability distribution for the leadtime, if at all. Kaplan (1970) is the earliest work to prove the optimality of an  $(s, S)$  inventory policy for a finite-horizon, periodic-review inventory system with multiple outstanding orders, stochastic leadtimes and a discounted total cost criterion. Order crossover is prohibited by assumption, so that an order placed at time  $t$  must arrive no later than the order placed at time  $t + 1$ . Ehrhardt (1984) extends this result to the infinite horizon. Song and Zipkin (1996) generalize these models (as well as Özekici and Parlar (1999)) by allowing the leadtime distribution to depend on an exogenous system that is modeled as a discrete-time Markov

chain (DTMC). When no fixed ordering cost is present, the optimality of a stationary, state-dependent basestock policy is proved for both the total expected discounted cost and long-run average cost models, where the basestock (or order up-to) levels *may* depend on the state of the exogenous system. While bounds on the optimal order-up-to levels are discussed, no explicit procedure is presented to determine optimal policy parameters, the long-run average cost for an arbitrary state-dependent basestock policy, nor the optimal long-run average cost.

In this paper we investigate an infinite-horizon periodic-review inventory model in which the probability distribution of the order leadtime is dependent on the state of a completely observed, exogenous Markov chain at the time of order placement, multiple orders may be outstanding at any given time, and order crossover cannot occur. Ordering costs are linear in the amount ordered, and stochastic demand that cannot be satisfied from on-hand inventory is fully backlogged. Since they are known to be optimal for this system, we consider only stationary, state-dependent basestock policies.

Using Markov reward theory, we develop an expression for the long-run average cost of an arbitrary stationary, state-dependent basestock policy. We then further restrict our attention to stationary, *state-invariant* basestock policies, in which the order-up-to levels are equivalent for all exogenous system states. We show how to explicitly calculate the optimal state-invariant basestock level and the long-run average cost and provide a sufficient condition for the optimality of a state-invariant basestock policy over all state-dependent basestock policies. We then apply the results to analyze the effects on a firm's inventory operations and long-run average cost of a possible major supply chain disruption (e.g. border closures). In the model, a domestic manufacturer orders a single product from a foreign supplier where the orders must cross an international border that is subject to closure due to security incidents, strikes, or other types of disruptions. We show that the optimal policy for the studied inventory system is a state-invariant basestock policy and present numerical results. These numerical results have important implications for the cooperation between business and government in disruption

management and contingency planning.

The rest of the paper is organized as follows. Section 2 gives the problem statement and preliminary results. In section 3, we develop the expression for the long-run average cost of an arbitrary stationary, state-dependent basestock policy. Restricting our attention to state-invariant basestock policies, we show how to calculate the optimal basestock level and long-run average cost and provide a sufficient condition for the optimality of a state-invariant basestock policy. The application to border closures and numerical results are presented in section 4. Finally, section 5 provides concluding remarks and areas of future research.

## 2 Problem Statement and Preliminaries

We consider an infinite-horizon, periodic-review inventory system in which a manufacturer periodically orders a single product from a supplier with unlimited supply (e.g. the supplier is always available). Order leadtimes are positive-valued and stochastic with probability distributions that are dependent on an exogenous supply system. At the beginning of each period, the inventory and supply system states are observed and an order, if any, is placed. The ordering cost is immediately incurred. Next, some subset of the outstanding orders arrive and demand is realized. Demand is stochastic and is satisfied from on-hand inventory if possible; otherwise, it is fully backlogged. Finally the on-hand inventory holding cost or the backorder penalty cost is assessed. The objective is to minimize the long-run average cost over the set of state-dependent basestock policies.

The manufacturer orders in discrete quantities (e.g. containers) at a cost of  $c$  per unit. Holding costs are  $h$  per unit per period for any inventory held. Penalty costs are  $p$  per unit per period for any backlogged demand. Let  $\hat{x}_t$  be the on-hand inventory at the end of period  $t$ , and define the

holding/penalty cost assessed at the end of period  $t$  to be

$$\hat{C}(x_t) = \begin{cases} -px_t & \text{if } x_t < 0 \\ hx_t & \text{if } x_t \geq 0. \end{cases} \quad (1)$$

Let  $D_t$  be a non-negative, integer random variable representing the demand in period  $t$ , where the demands in different periods are identically and independently distributed with probability mass function  $g$  and cumulative distribution function  $G$ . Demand is bounded such that  $D_t \in S_D = \{d_1, d_2, \dots, d_M\}$  where  $M < \infty$  and  $0 \leq d_1 < d_2 < \dots < d_M < \infty$ . Let  $D^{(l)}$  be the cumulative demand over  $l$  periods with probability mass function  $g_l$  and cumulative distribution function  $G_l$ .

The exogenous supply system is modeled by a discrete-time Markov chain  $\mathbf{I} = \{i_t, t \geq 0\}$ , where  $i_t$  represents the supply state at time  $t$ . This system is exogenous meaning that its evolution is independent of all other events. The Markov chain has state space  $S_I = \{1, 2, \dots, N < \infty\}$  and is time homogenous. For all  $t \geq 0$ , let  $p_{ij} = P(i_{t+1} = j | i_t = i)$  be the one-step transition probability from supply state  $i$  to  $j$  and let  $P_I = [p_{ij}]$  be the resulting stochastic matrix. Also for  $l \geq 0$ , define  $[P_I^l]_{ij} = p_{ij}^{(l)} = P(i_{t+l} = j | i_t = i)$  to be the  $l$ -step transition probability. Note that we do not make assumptions about the periodicity of  $\mathbf{I}$ . Since the chain has finite state space, let  $\pi^I$  be the unique stationary distribution. We use the subscript  $+$  to denote the next period, e.g.  $i_+$  instead of  $i_{t+1}$ .

To track outstanding orders through the supply system, each order is given a *position* attribute. Song and Zipkin (1996) present a general framework describing the concept of order positions and the transition dynamics from position to position (e.g. *order movement functions*). For example, the inventory positions can represent geographical locations or the number of periods that the order has been outstanding. Let  $\mathbf{z}_t = \{z_{kt}, 1 \leq k \leq K < \infty\}$  be the vector of outstanding orders where  $z_{kt}$  represents the cumulative order quantity in position  $k$  at time  $t$ . In addition to positions  $1 \leq k \leq K$ , we append position 0 to denote the current order and a dummy position  $\gamma$  to denote all orders that have arrived. We assume that for all  $t$ ,  $z_{0t}$  is a non-negative integer, and therefore for all  $k$  and  $t$ ,  $z_{kt}$

is a non-negative integer. The decision variables are described by the set  $\{z_{0t}, t \geq 0\}$ .

Let  $M(k|i)$  be the order movement function. Given  $i_t = i$ , the order currently in position  $k$  moves to position  $M(k|i)$  in period  $t + 1$  with probability 1. If  $M(k|i) = \gamma$ , then the order in position  $k$  has arrived. Thus given  $i_t = i$ ,

$$z_{k,t+1} = \sum_{\{n:M(n|i)=k\}} z_{nt}. \quad (2)$$

Given  $i_t = i$ , let  $M^l(k|i)$  be the random variable representing the position to which the order in position  $k$  will move at time  $t + l$ . The on-hand inventory at time  $t + 1$  given  $i_t = i$  is then

$$\hat{x}_{t+1} = \hat{x}_t + \sum_{\{k:M(k|i)=\gamma\}} z_{kt} - D_t. \quad (3)$$

The *inventory position* is the sum of all outstanding orders (prior to ordering) plus the on-hand inventory, and at time  $t$ , the inventory position is

$$x_t = \sum_{1 \leq k \leq K} z_{kt} + \hat{x}_t. \quad (4)$$

The random variable for the leadtime of the order placed at time  $t$  given  $i_t = i$  is

$$L(i) = \min_{l \geq 0} \{M^{l+1}(0|i) = \gamma\}. \quad (5)$$

As is standard practice, we make the key assumption that order crossover is prohibited. Formally we require  $P(L(i_+) \geq L(i) - 1) = 1$ , which is ensured by appropriately constructing the order movement functions (e.g.  $k \leq k' \Rightarrow M(k|i) \leq M(k'|i)$ ). We also assume that  $L(i)$  is finite with probability one.

Let  $S' = \{(i, \hat{x}, \mathbf{z}) \in S_I \times \mathbf{Z} \times \mathbf{Z}_+^K\}$  be the complete state space for each time period  $t \geq 0$ . A decision rule at time  $t$  is a function  $\delta_t : S' \rightarrow \mathbf{Z}^+$  that maps each possible state  $s \in S'$  at time  $t$  to a non-negative, integer-valued order quantity  $z_{0t}$ . Define a policy to be  $\Delta = \{\delta_t, t \geq 0\}$ . We will suppress subscripts and superscripts when appropriate, for example writing  $z_0$  for  $z_{0t}$ .

Since the average cost model does not discount future costs, future costs associated with the current order can be assessed to the period in which the order is placed. Recall that  $z_0 = \delta(i, \hat{x}, \mathbf{z})$ . From Song

and Zipkin (1996), the cost assessed to period  $t$  under policy  $\Delta$  is then

$$r_{\Delta}(i, \hat{x}, \mathbf{z}) = c\delta(i, \hat{x}, \mathbf{z}) + C(i, x + \delta(i, \hat{x}, \mathbf{z})), \quad (6)$$

where

$$C(i, x + \delta(i, \hat{x}, \mathbf{z})) = \sum_{l \geq 0} P(L(i) \leq l \leq L(i_+)) E \left[ \hat{C} \left( x + \delta(i, \hat{x}, \mathbf{z}) - D^{(l+1)} \right) \right]. \quad (7)$$

Note that  $C(i, x + z_0)$  is the expected cumulative holding and penalty costs incurred from the time the current order arrives until just before the order placed in the next period arrives.

For each starting state  $s = (i_0, \hat{x}_0, \mathbf{z}_0) \in S'$ , the total expected cost incurred from period 0 through period  $T - 1$  under policy  $\Delta$  is

$$v_T^{\Delta}(s) = E_s \left\{ \sum_{t=0}^{T-1} r_{\Delta}(i_t, \hat{x}_t, \mathbf{z}_t) \right\}, \quad (8)$$

and the average expected cost or *gain* of policy  $\Delta$  is

$$g^{\Delta}(s) = \lim_{T \rightarrow \infty} \frac{1}{T} v_T^{\Delta}(s) = \lim_{T \rightarrow \infty} \frac{1}{T} E_s \left\{ \sum_{t=1}^{T-1} r_{\Delta}(i_t, \hat{x}_t, \mathbf{z}_t) \right\}. \quad (9)$$

Under linear ordering costs, Song and Zipkin (1996) show that the average cost optimality equation is

$$g + h(i, x) = \min_{y \geq x} \{c(y - x) + C(i, y) + E[h(i_+, y - D)]\}, \quad (10)$$

where  $i$  is the state of the exogenous system,  $x$  is the inventory position, the order quantity is  $z_0 = y - x$  and  $g$  and  $h$  are respectively the gain and bias. The existence of an optimal policy that is a stationary, state-dependent basestock policy is then proved for both the total discounted expected cost and average cost models. That is, if  $\mathbf{y}^*$  is an optimal stationary, state-dependent basestock policy, then  $g^{\mathbf{y}^*} \leq g^{\mathbf{y}}$  for all stationary, state-dependent basestock policies  $\mathbf{y}$ . Therefore there exists an optimal policy  $\mathbf{y}^* = \{\delta^*, t \geq 1\}$  where the decision rules only require  $x_t, i_t$ , and a set of parameters,  $\{y^*(i), i \in S_I\}$  (the

basestock or order-up-to levels). The optimal ordering decision rule at time  $t$  is

$$\delta^*(x_t, i_t) = z_{0t} = \begin{cases} 0 & \text{if } x_t \geq y^*(i_t) \\ y^*(i_t) - x_t & \text{if } x_t < y^*(i_t). \end{cases} \quad (11)$$

Since only the state of the exogenous system and the inventory position are sufficient statistics, let  $S^y = S_I \times S_X^y$  be the sufficient state space where  $S_X^y$  is the state space of the inventory position. We use a superscript to show the dependence of the state spaces on the specific stationary, state-dependent basestock policy.

### 3 Arbitrary and State-Invariant Policies

We now present three primary results. Theorem 1 presents an expression for the long-run average cost of an arbitrary stationary, state-dependent basestock policy that is derived using Markov reward theory. Restricting our attention to state-invariant basestock policies, Theorem 2 shows how to calculate the optimal state-invariant order-up-to level and long-run average cost. Finally, in Corollary 1, we provide a sufficient condition for the optimality of a state-invariant basestock policy. We now present preliminary results useful to prove Theorem 1.

Let  $\mathbf{W} = \{W_t : t \geq 0\}$  be a Markov chain with countable or finite state space  $S$  and transition probability matrix  $P$ . Let  $r : S \rightarrow \Re$  be the cost function such that a cost of  $r(s)$  is incurred at time  $t$  when  $W_t = s$ . The bivariate stochastic process  $\{(W_t, r(W_t)) : t \geq 0\}$  is known as a Markov reward process (MRP). It is well known that in Markov decision processes (MDP), every stationary policy  $\Delta$  produces an MRP (denoted  $\mathbf{W}_\Delta$ ) with transition probability matrix  $P_\Delta$  and cost  $r_\Delta$  (see Puterman (1994)). This concept is central to our analysis. We will again suppress subscripts and superscripts when appropriate, for example writing  $P$  for  $P_\Delta$ .

Since there exists an optimal stationary, state-dependent basestock policy, we confine our interest to  $\Delta = \mathbf{y}$ . The resulting MRP,  $\mathbf{W}_y$ , has finite state space  $S^y = S_I \times S_X^y$ . Since demand is bounded

and due to the nature of the policy,  $S_X^y$  is a finite set with smallest element  $B_1 = \min_{i \in S_I} \{y(i)\} - d_M$  and largest element  $B_2 = \max_{i \in S_I} \{y(i)\} - d_1$ . The probability transition matrix is  $P_y$  and the cost assessed to period  $t$  is

$$r_y(i, x) = c(y(i) - x)^+ + C(i, x + (y(i) - x)^+), \quad (12)$$

where  $(z)^+ = \max\{z, 0\}$ .

Then for each  $s \in S$ , the average expected cost or gain of policy  $y$  is

$$g^y(s) = \lim_{T \rightarrow \infty} \frac{1}{T} v_T^y(s) = \lim_{T \rightarrow \infty} \frac{1}{T} E_s \left\{ \sum_{t=0}^{T-1} r_y(W_t) \right\} = \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} P_y^t r_y(s) = [P_y^* r_y](s), \quad (13)$$

where the limit exists since  $S$  is a finite set and where  $P^*$  is defined to be the *limiting matrix* of  $\mathbf{W}$ .

The limiting matrix is defined by the Cesaro limit (see Appendix A.4 in Puterman (1994)) to be

$$P^* = \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} P^t. \quad (14)$$

Regardless of the periodicity characteristics of  $\mathbf{W}$ , the Cesaro limit exists for both countable- and finite-state Markov chains (and is equivalent to the regular limit if the chain is aperiodic). Furthermore if the Markov chain is irreducible and positive recurrent (which is true under our assumptions), then a unique stationary distribution  $\pi$  solves the system of equations  $\pi = \pi P$  subject to  $\sum_{s \in S} \pi_s = 1$  and  $\pi_s \geq 0$  for all  $s \in S$ . A property of the limiting matrix is that  $P^* P = P^*$ . Therefore since the stationary distribution is unique,  $P^* = \pi^T e^T$  where  $e$  is a column vector of ones. That is, the rows of  $P^*$  are identical and are each equivalent to the stationary distribution  $\pi$ . Finally, since  $\mathbf{W}$  has finite state space and is irreducible, the gain is constant for all  $s \in S$  and is

$$g^y = [P_y^* r_y] = \pi_y r_y = \sum_{(i,x) \in S_y} \pi_{(i,x)}^y [c(y(i) - x)^+ + C(i, x + (y(i) - x)^+)]. \quad (15)$$

We now present a key lemma that will be used in the proof of Theorem 1.

**Lemma 1.**  $\sum_{(i,x) \in S} \pi_{(i,x)} (y(i) - x)^+ = E[D]$ .

*Proof.* We note that for all  $t \geq 0$ ,  $x_{t+1} = x_t + (y(i_t) - x_t)^+ - D_t$ . It follows that

$$\lim_{t \rightarrow \infty} E \left[ (y(i_t) - x_t)^+ \right] = \lim_{t \rightarrow \infty} E[x_{t+1}] - \lim_{t \rightarrow \infty} E[x_t] + \lim_{t \rightarrow \infty} E[D_t], \quad (16)$$

where  $E$  is the expectation operator conditioned on  $(i_0, x_0)$  and the limit is the Cesaro limit. For any function  $f$ ,

$$\lim_{t \rightarrow \infty} E[f(i_t, x_t)] = \sum_{(i,x) \in S} f(i, x) \pi_{(i,x)}.$$

Note, therefore, that

$$\begin{aligned} \lim_{t \rightarrow \infty} E \left[ (y(i_t) - x_t)^+ \right] &= \sum_{(i,x) \in S} (y(i) - x)^+ \pi_{(i,x)}, \\ \lim_{t \rightarrow \infty} E[x_t] &= \sum_{(i,x) \in S} x \pi_{(i,x)}, \\ \lim_{t \rightarrow \infty} E[D_t] &= E[D], \end{aligned}$$

where the last equality follows from the fact that the  $\{D_t\}$  are independent and identically distributed.

We note that

$$\begin{aligned} \lim_{t \rightarrow \infty} E[x_{t+1}] &= \lim_{t \rightarrow \infty} E[E[x_{t+1} | i_t, x_t]] \\ &= \sum_{(i,x) \in S} E[x' | i, x] \pi_{(i,x)}, \end{aligned}$$

where  $E[x' | i, x] = \sum_{(i',x') \in S} x' [P]_{(i,x)(i',x')}$ . Thus,

$$\begin{aligned} \lim_{t \rightarrow \infty} E[x_{t+1}] &= \sum_{(i,x) \in S} \sum_{(i',x') \in S} x' [P]_{(i,x)(i',x')} \pi_{(i,x)} \\ &= \sum_{(i',x') \in S} x' \sum_{(i,x) \in S} \pi_{(i,x)} [P]_{(i,x)(i',x')} \\ &= \sum_{(i',x') \in S} x' \pi_{(i',x')}, \end{aligned}$$

where the next to last equality follows from the fact that  $\pi = \pi P$ . Collecting terms into equation (16) produces the result.  $\square$

The following theorem provides an expression used to calculate the long-run average cost of an arbitrary stationary, state-dependent basestock policy. This expression allows for direct cost comparisons

between different stationary, state-dependent basestock policies. We use superscripts to emphasize the dependence of the state-space,  $S^y$ , and the stationary distribution,  $\pi^y$ , on the specific policy  $y$ .

**Theorem 1.** *Let  $y$  be any stationary, state-dependent basestock policy whose resulting MRP has state-space  $S^y$  and stationary distribution  $\pi^y$ . Then*

$$g^y = cE[D] + \sum_{(i,x) \in S^y} \pi_{(i,x)}^y C(i, x + (y(i) - x)^+). \quad (17)$$

*Proof.* From equation (15) and Lemma 1, we have

$$\begin{aligned} g^y &= \sum_{(i,x) \in S^y} \pi_{(i,x)}^y [c(y(i) - x)^+ + C(i, x + (y(i) - x)^+)] \\ &= cE[D] + \sum_{(i,x) \in S^y} \pi_{(i,x)}^y C(i, x + (y(i) - x)^+). \end{aligned}$$

□

Lemmas 2 and 3 given below are used in the proof of Theorem 2.

**Lemma 2.** *For all  $i \in S_I$ ,  $C(i, y)$  is convex in  $y$  and  $\lim_{|y| \rightarrow +\infty} C(i, y) = +\infty$ .*

*Proof.* The convexity of  $C(i, y)$  follows from the convexity of  $\hat{C}(x)$  and the definition of  $C(i, y)$ . The proof of the second part follows a similar proof as that of Lemma 2 in Song and Zipkin (1993). □

**Lemma 3.** *For all  $i \in S_I$  and  $y$ ,*

$$\Delta C(i, y) \equiv C(i, y + 1) - C(i, y) = (p + h) \sum_{l \geq 0} P(L(i) \leq l \leq L(i_+)) G_{l+1}(y) - p\delta_i,$$

where  $\delta_i \equiv \sum_{l \geq 0} P(L(i) \leq l \leq L(i_+))$ .

*Proof.* It follows from equation (1) that

$$\Delta \hat{C}(y - d) \equiv \hat{C}(y + 1 - d) - \hat{C}(y - d) = \begin{cases} -p & \text{if } d > y \\ h & \text{if } d \leq y. \end{cases}$$

Then from Lemma 2,

$$\begin{aligned}
\Delta C(i, y) &= \sum_{l \geq 0} P(L(i) \leq l \leq L(i_+)) E \left[ \Delta \hat{C} \left( y - D^{(l+1)} \right) \right] \\
&= \sum_{l \geq 0} P(L(i) \leq l \leq L(i_+)) \sum_{d=d_1}^{(l+1)d_M} g_{l+1}(d) \Delta \hat{C} \left( y - D^{(l+1)} \right) \\
&= \sum_{l \geq 0} P(L(i) \leq l \leq L(i_+)) \left( \sum_{d=d_1}^y g_{l+1}(d) h + \sum_{d=y+1}^{(l+1)d_M} g_{l+1}(d) (-p) \right) \\
&= \sum_{l \geq 0} P(L(i) \leq l \leq L(i_+)) \left( h \sum_{d=d_1}^y g_{l+1}(d) - p \left( 1 - \sum_{d=d_1}^y g_{l+1}(d) \right) \right) \\
&= \sum_{l \geq 0} P(L(i) \leq l \leq L(i_+)) \left( (p+h) \left( \sum_{d=d_1}^y g_{l+1}(d) \right) - p \right) \\
&= \sum_{l \geq 0} P(L(i) \leq l \leq L(i_+)) ((p+h) G_{l+1}(y) - p) \\
&= (p+h) \sum_{l \geq 0} P(L(i) \leq l \leq L(i_+)) G_{l+1}(y) - p \delta_i,
\end{aligned}$$

where the expectation operator in the first equation is with respect to  $D^{(l+1)}$ .  $\square$

Assume that we now restrict the set of feasible stationary, state-dependent basestock policies to those policies  $\hat{y}$  where  $y(\hat{0}) = y(\hat{1}) = \dots = y(\hat{N}) \equiv \hat{y}$ . We refer to  $\hat{y}$  as a stationary state-invariant basestock policy and  $\hat{y}$  as the state-invariant basestock or order-up-to level. While  $\hat{y}$  may be suboptimal, a firm may desire an inventory policy with this form for simplicity. Theorem 2 shows how to calculate the optimal state-invariant basestock level and long-run average cost. Let  $\hat{y}^*$  denote the smallest among all optimal state-invariant basestock levels.

**Theorem 2.**

(i) If

$$\tilde{y} = \min \left\{ d_1 \leq y < \infty : y \in Z, \sum_{l \geq 0} G_{l+1}(y) \sum_{i \in S_I} \pi_i^I \frac{P(L(i) \leq l \leq L(i_+))}{\delta_i} \geq \frac{p}{p+h} \right\}, \quad (18)$$

then  $\tilde{y} = \hat{y}^*$ .

(ii)  $g^{\hat{y}^*} = cE[D] + \sum_{i \in S_I} \pi_i^I C(i, \hat{y}^*)$ .

*Proof.* By definition,  $\hat{y}^* = \min \{ \operatorname{argmin}_{y: y \in Z} \{ \sum_{i \in S_I} \pi_i^I C(i, y) \} \}$ . Since  $L(i)$  is finite with probability 1 and order crossover is prohibited,  $\delta_i > 0$  for all  $i \in S_I$ . From Lemma 3, for all  $i \in S_I$ , if  $y < 0$  then  $\Delta C(i, y) = -p\delta_i < 0$  since  $G_{l+1}(y) = 0$  for  $y < d_1$  and  $\delta_i > 0$ . It follows from Lemma 2 that  $\hat{y}^*$  is finite. We can rewrite the definition of  $\hat{y}^*$  with the new bounds and two necessary conditions for optimality as  $\hat{y}^* = \min \{ d_1 \leq y \leq \infty : y \in Z, \sum_{i \in S_I} \pi_i^I \Delta C(i, y) \geq 0 \}$ . The result then follows from Lemma 3. Part (ii) follows from Theorem 1.

$$\begin{aligned}
g^{\hat{y}^*} &= cE[D] + \sum_{(i,x) \in S^{\hat{y}^*}} \pi_{(i,x)}^{\hat{y}^*} C(i, x + (\hat{y}^* - x)^+) \\
&= cE[D] + \sum_{i \in S_I} \sum_{x \in S_X^{\hat{y}^*}} \pi_{(i,x)}^{\hat{y}^*} C(i, \hat{y}^*) \\
&= cE[D] + \sum_{i \in S_I} \pi_i^I C(i, \hat{y}^*).
\end{aligned}$$

It can be shown that  $\pi_{(i,x)}^{\hat{y}^*} = \pi_i^I P(D = \hat{y}^* - x)$ . The last equality holds by the law of total probability. □

As in Song and Zipkin (1996), define the *myopic* cost function  $H(i, y) = cE[D] + C(i, y)$  and let  $y^+(i)$  denote the smallest among all minimizers of  $H(i, y)$  (known as myopic order-up-to levels). Also considering *all* stationary, state-dependent basestock policies, let  $y^*(i)$  denote the smallest among all unrestricted optimal order-up-to levels for exogenous system state  $i$ .

**Corollary 1.**

(i) Let  $\tilde{i} = \min \{ \operatorname{argmin}_i \{ y^+(i) \} \}$ . Then for all  $i \in S_I$ ,  $y^+(\tilde{i}) \leq y^*(i) \leq y^+(i)$ .

(ii) For each  $i \in S_I$ , if

$$\tilde{y} = \min \left\{ d_1 \leq y < \infty : y \in Z, \sum_{l \geq 0} G_{l+1}(y) \frac{P(L(i) \leq l \leq L(i_+))}{\delta_i} \geq \frac{p}{p+h} \right\}, \quad (19)$$

then  $\tilde{y} = y^+(i)$ .

(iii) If  $y^+(0) = y^+(1) = \dots = y^+(N) \equiv y^+$ , then  $y^+ = y^*(0) = y^*(1) = \dots = y^*(N) = \hat{y}^*$  and

$$g^{\hat{y}^*} = cE[D] + \sum_{i \in S_I} \pi_i^I C(i, \hat{y}^*) = g^*, \quad (20)$$

where  $g^*$  is the minimal gain over *all* stationary, state-dependent basestock policies.

*Proof.* Part (i) restates Theorem 3(a) and 3(b) in Song and Zipkin (1996). The proof of part (ii) follows a similar proof as that of Theorem 2(i) in this paper. In part (iii), the optimality of  $y^+$  follows directly from part (i). The left equality in (20) holds by Theorem 2(ii) and the right equality follows from Theorem 1 under the unrestricted optimal policy  $\hat{y}^*$ .  $\square$

## 4 Border Closures without Congestion

Consider a supply chain consisting of a foreign supplier and a domestic manufacturer. Orders are shipped on a fixed transportation route from the supplier to a domestic port of entry for importation (e.g. a seaport or land border); the transit time is  $L$  periods. Assume that the inland transportation time between the port of entry and the manufacturer is negligible (a simple modification in the model could allow non-zero inland time). Upon arrival at the port of entry, if the border is open then the order arrives to the manufacturer without delay. Otherwise, the order is held at the port of entry until the border reopens. When the border reopens, all orders arriving to, or currently waiting at, the border gain entry and arrive at the manufacturer. In the analysis that follows, we assume that congestion at the border resulting from the accumulation of orders during periods of border closure is negligible and has no effect on order leadtimes.

We can now describe this supply system with the following DTMC model. Let the state space be  $S_I = \{O, C\}$  where  $i_t = O$  indicates that the border is open in period  $t$  and  $i_t = C$  indicates that it is

closed. The transition probability matrix is

$$P_I = \begin{pmatrix} 1 - p_{OC} & p_{OC} \\ p_{CO} & 1 - p_{CO} \end{pmatrix},$$

where we assume that  $0 < p_{OC} < 1$  and  $0 < p_{CO} < 1$ , since the extreme values result in uninteresting systems. For  $l \geq 0$ , it is well known that the  $l$ -step transition probability matrix for a two-state Markov chain with state space  $S_I = \{O, C\}$ ,  $0 < p_{OC} < 1$ , and  $0 < p_{CO} < 1$  is given by

$$P_I^l = (p_{OC} + p_{CO})^{-1} \left\{ \begin{pmatrix} p_{CO} & p_{OC} \\ p_{CO} & p_{OC} \end{pmatrix} + (1 - p_{OC} - p_{CO})^l \begin{pmatrix} p_{OC} & -p_{OC} \\ -p_{CO} & p_{CO} \end{pmatrix} \right\}. \quad (21)$$

The stationary distribution of this chain is

$$\pi^I = \{\pi_O^I, \pi_C^I\} = \left\{ \frac{p_{CO}}{p_{OC} + p_{CO}}, \frac{p_{OC}}{p_{OC} + p_{CO}} \right\}.$$

In this model, the component  $z_{kt}$  of the order vector represents the order that has been outstanding for  $k$  time periods at period  $t$  for  $k = \{0, 1, 2, \dots, L-1\}$ . Since orders may accumulate at the border when it is closed,  $z_{Lt}$  represents the sum of all orders that have been outstanding for *at least*  $L$  periods.

The order movement function describing this system is given by:

$$M(k|O) = \begin{cases} k+1 & \text{if } 0 \leq k < L \\ \gamma & \text{if } k = L \end{cases}$$

$$M(k|C) = \begin{cases} k+1 & \text{if } 0 \leq k < L \\ L & \text{if } k = L. \end{cases}$$

This order movement function prevents crossover.

Let  $\mathbf{W} = \{W_t \equiv (i_t, x_t) : t \geq 0\}$  be the Markov chain on state space  $S = S_I \times S_X$  that arises under the stationary, state-dependent basestock policy  $\mathbf{y}$ . Suppose that  $\mathbf{W}$  has transition probability matrix  $P$ . Assuming that  $y^*(O) = y^*(C) \equiv \hat{y}$  (we will show this to be true in Corollary 2), the one-step transition probability of  $\mathbf{W}$  is  $[P]_{(i,x),(j,x')} = p_{ij}P(D = \hat{y} - x')$  for all  $(i, x) \in S$  and  $(j, x') \in S$ .

We now develop the probability distribution for the order leadtime  $L(i)$ . An order placed at time  $t$  when  $i_t = i$  will arrive at time  $t + L(i)$ . From the order movement function,  $P(L(i) = m) = 0$  for  $0 \leq m \leq L - 1$ . The leadtime is exactly  $L$  if and only if  $i_{t+L} = O$  and so  $P(L(i) = L) = p_{iO}^{(L)}$ . Similarly, the leadtime is exactly  $L + 1$  if and only if  $i_{t+L} = C$  and  $i_{t+L+1} = O$ . Therefore  $P(L(i) = L + 1) = p_{iC}^{(L)} p_{CO}$ . Note that  $P(L(i) = L + 1) \neq p_{iO}^{(L+1)}$  since  $i_{t+L}$  cannot be  $O$ . Similarly for  $m \geq 2$ ,  $P(L(i) = L + m) = p_{iC}^{(L)} p_{CC}^{m-1} p_{CO}$ . In summary,

$$P(L(i) = l) = \begin{cases} 0 & \text{if } l < L \\ p_{iO}^{(L)} & \text{if } l = L \\ p_{iC}^{(L)} p_{CC}^{l-L-1} p_{CO} & \text{if } l > L. \end{cases} \quad (22)$$

It can then be shown that

$$P(L(i) \leq l) = \begin{cases} 0 & \text{if } l < L \\ p_{iO}^{(L)} & \text{if } l = L \\ 1 - p_{iC}^{(L)} p_{CC}^{l-L} & \text{if } l > L, \end{cases} \quad (23)$$

$$P(L(i) \leq l \leq L(i_+)) = P(L(i) \leq l) - \sum_{j \in S_I} p_{ij} P(L(j) \leq l - 1) = \begin{cases} 0 & \text{if } l < L \\ p_{iO}^{(L)} & \text{if } l = L \\ p_{iO}^{(L)} p_{OC} p_{CC}^{l-L-1} & \text{if } l > L, \end{cases} \quad (24)$$

and

$$\delta_i = p_{iO}^{(L)} \left( 1 + \frac{p_{OC}}{p_{CO}} \right). \quad (25)$$

We now present the following corollary which states that the optimal stationary, state-dependent base-stock policy for the border closure model without congestion is actually a stationary, state-invariant basestock policy. We note that even though the optimal order-up-to levels are independent of the exogenous system state, it is not valid to claim that the model then reduces to one with a single-state exogenous border system. The border system clearly affects the leadtime probability distribution, the

order-up-to levels and the resultant long-run average cost.

**Corollary 2.** *For the border closure model without congestion,  $y^*(O) = y^*(C)$ .*

*Proof.* Since we assume  $0 < p_{OC} < 1$  and  $0 < p_{CO} < 1$ , it is easy to show from equation (21) that  $\delta_i > 0$  for all  $i \in S_I$ . Consider the left-hand side of the inequality in the second necessary condition within the minimization in Corollary 1(ii). From equations (24) and (25), this expression can be written as

$$\sum_{l \geq 0} \frac{P(L(i) \leq l \leq L(i_+))}{\delta_i} G_{l+1}(y) = \frac{G_{L+1}(y)}{1 + \frac{p_{OC}}{p_{CO}}} + \sum_{l > L} \left( \frac{p_{OC} p_{CC}^{l-L-1}}{1 + \frac{p_{OC}}{p_{CO}}} \right) G_{l+1}(y),$$

which is independent of  $i$ . Thus the same  $\tilde{y}$  will be found in Corollary 1(ii) for both border states.  $\square$

We now construct an example supply chain for a computational study. Consider an international supply chain subject to border closures where a manufacturer based in the Western United States orders a single product from a single foreign supplier in Asia. Orders are measured in units of container loads and are placed each day. The containers are shipped by ocean carrier and the minimum leadtime from supplier to the domestic seaport is  $L = 15$  days, which is consistent with typical carrier services from Asia to the Western United States. The economic parameters are  $c = \$150,000$ ,  $h = \$100$ , and  $p = \$1,000$ . Demand has a truncated Poisson distribution with mean demand of 0.5 container per day and a maximum realizable demand in any period of  $d_M = 10$ . A truncated Poisson distribution assigns Poisson probabilities to all demand realizations up through  $d_{M-1}$  and a probability of  $1 - G(d_{M-1})$  to  $d_M$  where  $d_M$  is chosen such that  $1 - G(d_{M-1}) < \epsilon$  for some  $\epsilon > 0$ . In our example,  $P(D = 10) = 1 - G(9) = 1.63 \times 10^{-10}$ .

Results for this example are depicted in the figures. Figure 1 presents the optimal long-run average cost for a wide range of border transition probabilities while Figure 2 shows the optimal order-up-to level, which is state-invariant (e.g.  $y^* = y^*(O) = y^*(C)$ ). In general, the optimal long-run average cost and order-up-to level are more sensitive to  $p_{CO}$  than to  $p_{OC}$ . These two probabilities offer different

measures of border closure severity, respectively, likely duration and occurrence likelihood. If the border is in state  $i$ , then the expected number of periods until the border transitions to state  $j$  is  $1/p_{ij}$ . Therefore the expected duration of a border closure ( $1/p_{CO}$ ) more negatively affects a firm's productivity as measured by cost and inventory than the probability of a border closure ( $p_{OC}$ ). Note also that the greatest increases in average cost and order-up-to level occur when  $p_{CO}$  is small (e.g. large expected closures). Thus, while prevention of disruptions is important, it is critical that business engage and cooperate with government to design effective contingency plans that reduce the duration of a disruption and quickly return the system to a normal state of operation.

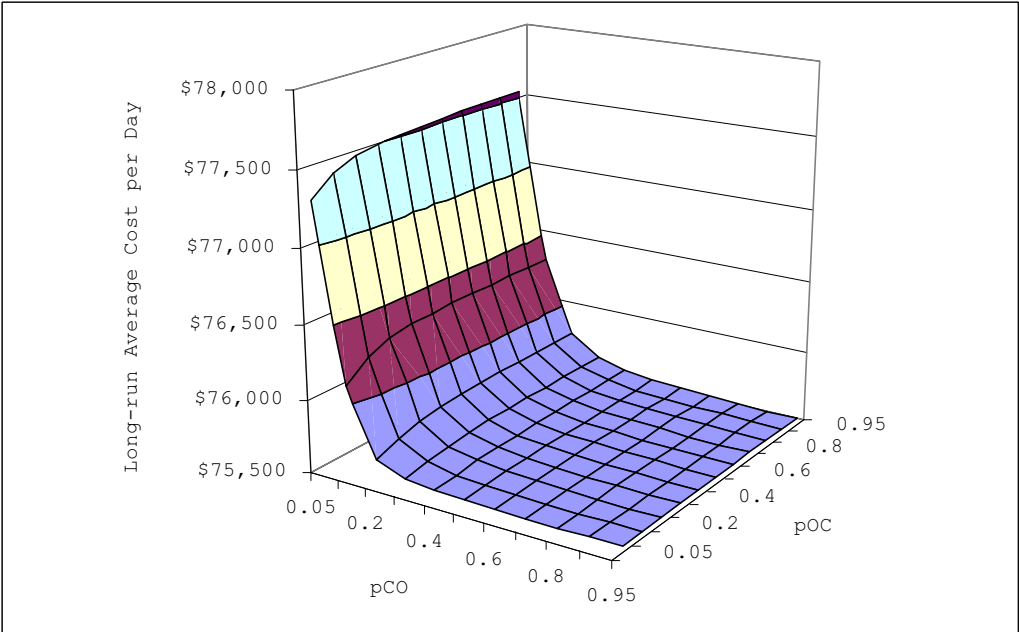


Figure 1: Long-run Average Cost vs.  $p_{OC}$  and  $p_{CO}$ .

Border closures are typically considered to be rare-events and are therefore not included in regular operational planning models. Suppose that a firm optimizes its inventory policy without explicitly modeling border closures and implements them in a real-world environment in which the border may experience closures. Clearly the firm's inventory policy will be sub-optimal, but it is interesting to investigate how poor this policy might be. To address this question, we develop an optimal inventory

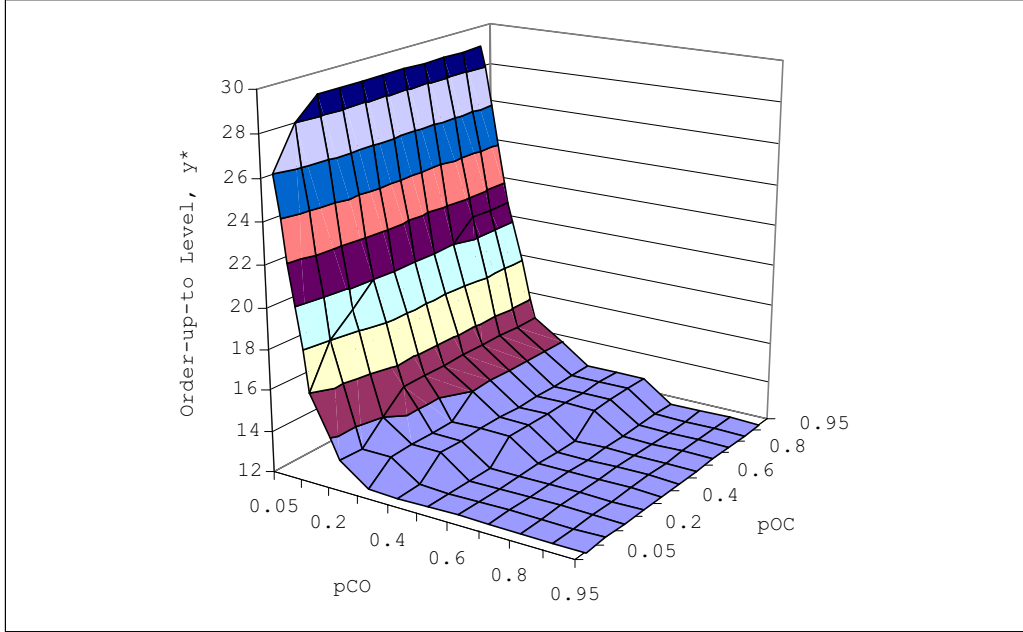


Figure 2: Order-up-to Level vs.  $p_{OC}$  and  $p_{CO}$ .

policy for the system described above using a model in which  $p_{OC} = 0$  and therefore ignores the risk of border closures, and compare its long-run average daily cost performance to optimal policies when the probability of border closure is nonzero. Figure 3 shows the increase in average cost per day resulting from the use of the sub-optimal policy instead of the optimal policy for a range of border transition probabilities. For example, when  $p_{OC} = 0.01$  and  $p_{CO} = 0.05$ , the increase in long-run average cost is \$1,080 per day, or \$394,352 per year. We see a precipitous increase in the additional long-run average cost as  $p_{CO}$  approaches zero.

Fixing  $p_{OC} = 0.01$  and  $p_{CO} = 0.1$ , we vary the minimum leadtime and graph the optimal results in Figure 4. As expected, the optimal order-up-to level and the long-run average cost increase as the minimum leadtime increases. The rate of increase in both order-up-to level and the long-run average cost decreases as the minimum leadtime increases until it appears to become constant. These results re-enforce the conventional wisdom that shorter leadtimes are preferable.

From Theorem 2(i), we see that the optimal order-up-to level depend on the cost parameters only

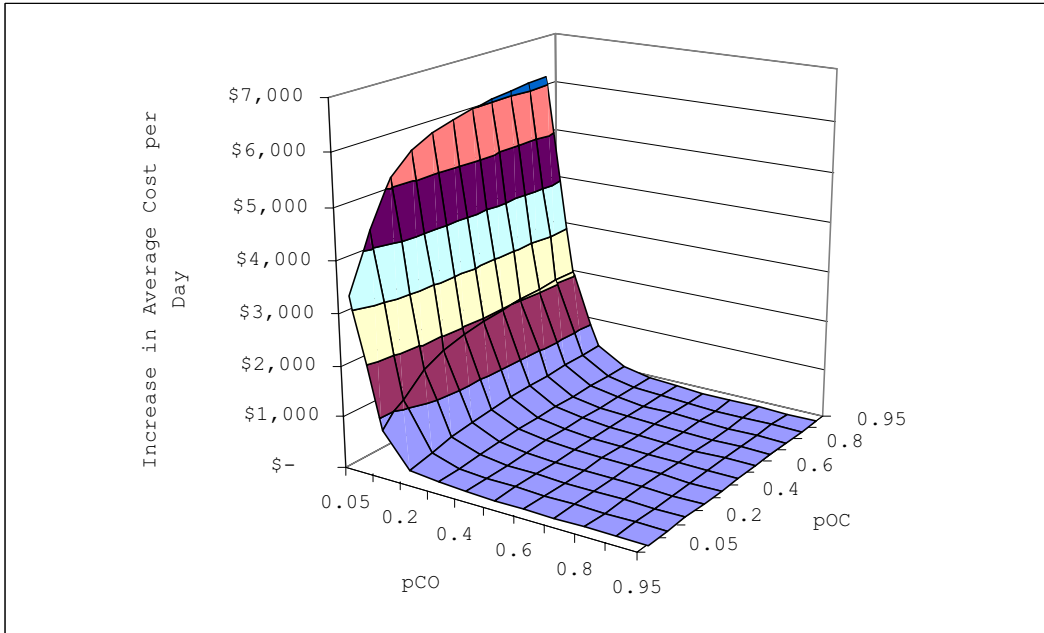


Figure 3: Increase in Long-run Average Cost vs.  $pOC$  and  $pCO$ .

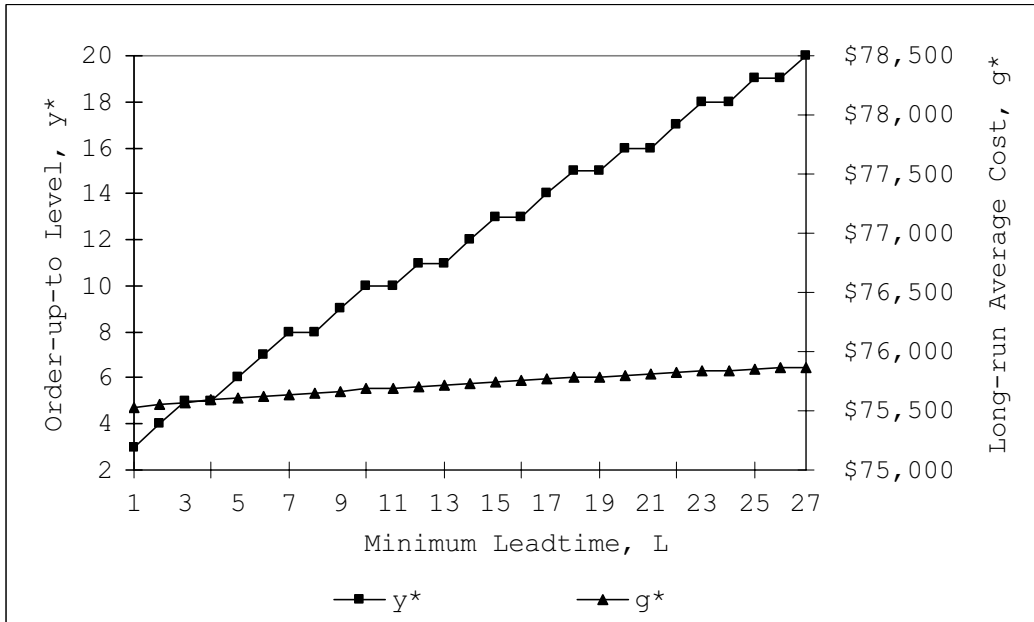


Figure 4: Order-up-to Level and Long-run Average Cost vs.  $L$ .

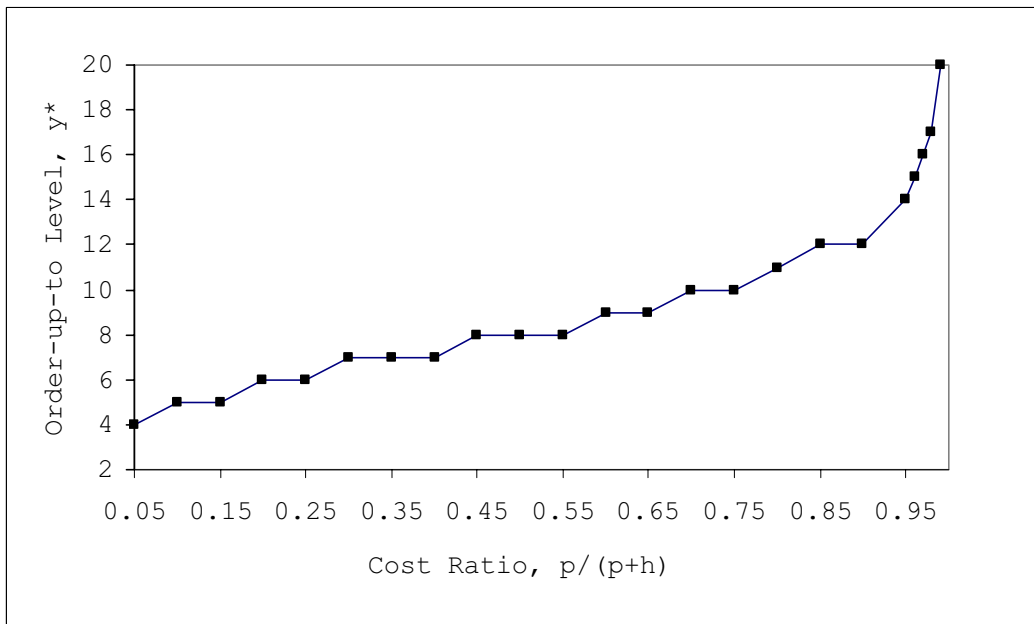


Figure 5: Order-up-to Level vs. Cost Ratio,  $p/(p+h)$ .

through the ratio  $p/(p+h)$ . Fixing  $p_{OC} = 0.01$ ,  $p_{CO} = 0.1$ , and  $L = 15$ , we vary this ratio and graph the optimal order-up-to level in Figure 5. Ratio values less than 0.5 imply that  $h > p$  and ratio values greater than 0.5 imply  $p > h$ . It is interesting that the optimal order-up-to level increases linearly with the cost ratio until the cost ratio approaches 1 (i.e. a range of 0 to approximately 0.95, which accounts for a wide range of penalty costs from 0 to 20 times the holding cost). The rapid increase in order-up-to level as the ratio approaches 1 has two interpretations. The first interpretation considers a large penalty cost relative to the holding cost. Let  $\alpha$  denote the cost ratio. Then for a given value of  $\alpha$ , it is easily shown that  $p = \left(\frac{\alpha}{1-\alpha}\right)h$ . As  $\alpha$  approaches 1 with a fixed holding cost, the penalty cost approaches infinity. This results in an increased order-up-to level that protects against costly backorders. The second interpretation considers a small holding cost relative to the penalty cost. Noting that  $h = \left(\frac{1-\alpha}{\alpha}\right)p$ , as  $\alpha$  approaches 1 with a fixed penalty cost, the holding cost approaches 0. Since the holding cost is small, the order-up-to level can be increased to reduce the risk of backorders without incurring large holding costs.

There are many combinations of  $h$  and  $p$  for each value of the cost ratio and we therefore examine the effects of each cost separately on the order-up-to level and long-run average cost. We vary the holding cost from while fixing  $p = \$1,000$  and then vary the penalty cost while while fixing  $h = \$100$ . These studied ranges for  $h$  and  $p$  correspond to approximately the same range of cost ratio values, respectively  $[0.91,0.33]$  and  $[0.5,0.96]$ . Figures 6 and 7 display the results. As expected, the results are consistent with those obtained by varying the cost ratio: increasing  $p$  while fixing  $h$  increases the order-up-to level while increasing  $h$  while fixing  $p$  decreases the order-up-to level. The long-run average cost increases with both  $h$  and  $p$ . While the costs in both studies exhibit decreasing rates of increase as the holding or penalty cost increases, the rate diminishes more quickly with the penalty cost than with the holding cost. As a result, the difference in the long-run average cost over the studied range of  $h$  is greater than that over the studied range of  $p$ . This indicates that over a large range of the cost ratio (i.e.  $\alpha \leq 0.95$ ), the long-run average cost is more sensitive to changes in the holding cost than to changes in the penalty cost.

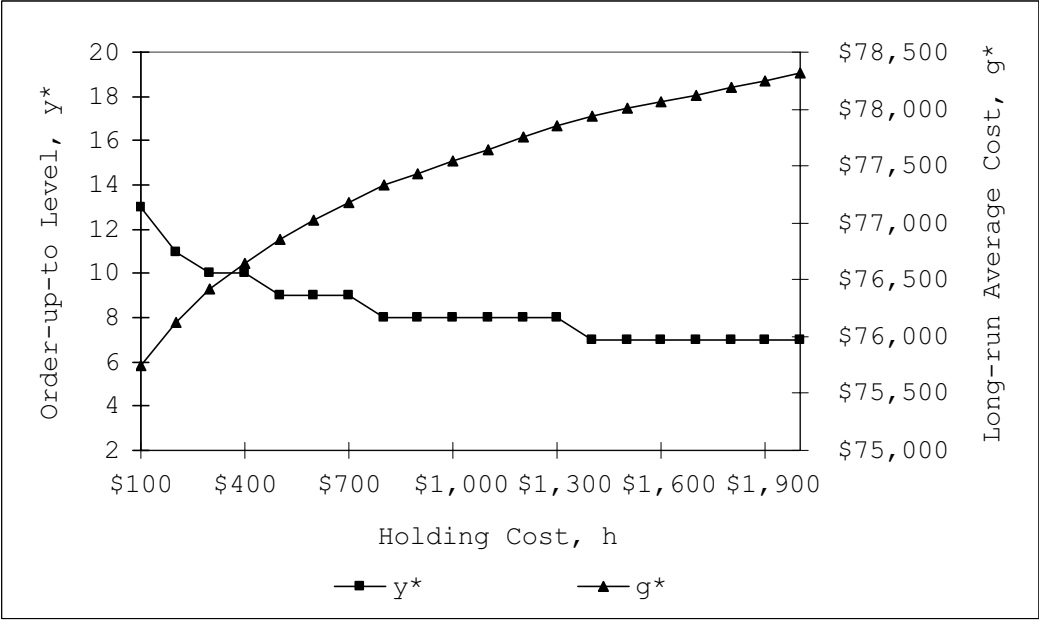


Figure 6: Order-up-to Level and Long-run Average Cost vs.  $h$ .

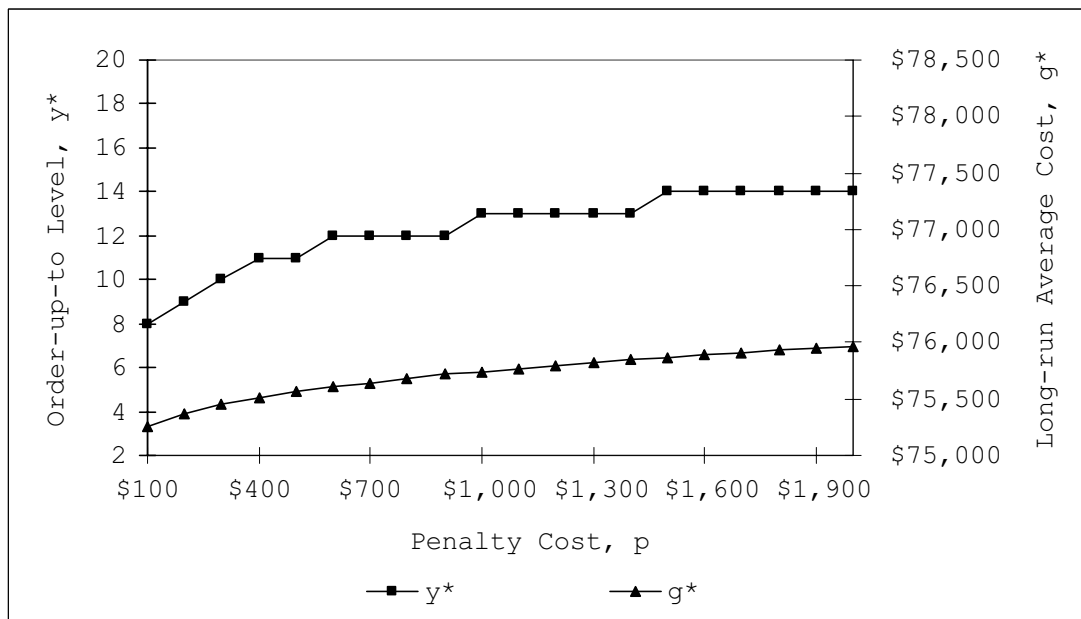


Figure 7: Order-up-to Level and Long-run Average Cost vs.  $p$ .

## 5 Conclusions

In this paper, we extend the inventory literature by deriving an expression for the long-run average cost under an arbitrary stationary, state-dependent basestock policy for an inventory system in which order leadtimes are dependent on an exogenous system and ordering costs are linear in the amount ordered. We show how to explicitly calculate an optimal stationary, state-invariant basestock policy and the associated long-run average cost and provide a sufficient condition for the optimality of a stationary, state-invariant basestock policy.

We then apply this model to an inventory control problem where an important and timely supply chain disruption, e.g. border closures, may occur. We develop the probability distribution for order leadtimes when border congestion is negligible and show that a state-invariant basestock policy is optimal, in which the optimal order-up-to levels are equivalent for the open and closed states. We present numerical results for an example supply chain system that show how the optimal order-up-to levels and the long-run average cost are affected by the border transition probabilities, the minimum

leadtime, and the economic parameters and we examine the increase in long-run average cost resulting from the use of a sub-optimal policy. Our results indicate that the optimal inventory policy and long-run average cost are more sensitive to the expected duration of a disruption than to the likelihood of the disruption occurring. Since borders (e.g. seaports, land borders, and airports) are generally publicly owned and operated and utilized by the private sector, these results have important management and policy implications for business and government. Governments tend to allocate a large portion of their resources to prevention. While clearly the prevention of a disruption is critically important, our results support business and government engaging and cooperating to reduce the duration of border closures through effective disruption management and contingency planning.

In Theorem 2, we restricted the set of feasible stationary, state-dependent basestock policies to stationary, state-invariant basestock policies. A subject of future research is an examination of the sub-optimality of state-invariant basestock policies in comparison to unrestricted state-dependent basestock policies as well as a continued investigation into necessary and/or sufficient conditions for the optimality of state-invariant basestock policies. Extensions to the border closure model include expanding the border system state-space to more than two border states (for example, the states would represent increasing levels of risk of a border closure) and incorporating the border congestion that results from the accumulation of orders during periods of border closure. Motivated by the 2002 seaport labor strike in Western United States and the possibility of re-routing freight through Canadian or Mexican ports, another area for future research is the study of a network model that incorporates the re-routing of freight to other ports of entry when a set of desired ports of entry close.

## **Acknowledgements**

This ongoing research is supported by The Logistics Institute-Asia Pacific, a collaboration between the Georgia Institute of Technology and the National University of Singapore and the ATLANTIC Project,

a research partnership between the ITS Joint Program Office of USDOT, the Canadian Transportation Ministry, and the European Commission. Brian Lewis is supported by a Dwight D. Eisenhower Fellowship for Graduate Study in Transportation from the National Highway Institute of the USDOT.

## References

- [1] Arreola-Risa, A. and G. DeCroix, "Inventory Management under Random Supply Disruptions and Partial Backorders," *Naval Research Logistics*, 45 (1998), 687-703.
- [2] Bonner, R. C., "Testimony of Commissioner Robert C. Bonner, U.S. Customs and Border Protection Before the National Commission on Terrorist Attacks Upon the United States, January 26, 2004," U.S. Customs and Border Protection, Washington, D.C., 2004.
- [3] Ehrhardt, R., " $(s, S)$  Policies for a Dynamic Inventory Model with Stochastic Lead Times," *Operations Research*, 32 (1984), 121-132.
- [4] Gerencser, M., J. Weinberg, and D. Vincent, "Port Security War Game: Implications for U.S. Supply Chains," Booz Allen Hamilton, Inc., 2003.
- [5] Hendricks, K. B. and V. R. Singhal, "The Effect of Supply Chain Glitches on Shareholder Value", *Journal of Operations Management*, 21 (2003), 501-522.
- [6] Kaplan, R., "A Dynamic Inventory Model with Stochastic Lead Times," *Management Science*, 16 (1970), 491-507.
- [7] Moinzadeh, K. and P. Aggarwal, "Analysis of a Production/Inventory System Subject to Random Disruptions," *Management Science*, 43 (1997), 1577-1588.
- [8] Özekici, S. and M. Parlar, "Inventory Models with Unreliable Suppliers in a Random Environment," *Annals of Operations Research*, 91 (1999), 123-136.

- [9] Parlar, M., “Continuous-review Inventory Problem with Random Supply Interruptions,” *European Journal of Operational Research*, 99 (1997), 366-385.
- [10] Parlar, M. and D. Berkin, “Future Supply Uncertainty in EOQ Models,” *Naval Research Logistics*, 38 (1991), 107-121.
- [11] Parlar, M. and D. Perry, “Analysis of a  $(Q, r, T)$  Inventory Policy with Deterministic and Random Yield When Future Supply is Uncertain,” *European Journal of Operational Research*, 84 (1995), 431-443.
- [12] Parlar, M., Y. Wang, and Y. Gerchak, “A Periodic Review Inventory Model with Markovian Supply Availability,” *International Journal of Production Economics*, 42 (1995), 131-136.
- [13] Puterman, M., *Markov Decision Processes: Discrete Stochastic Dynamic Programming*. John Wiley, New York, 1994.
- [14] Sheffi, Y., “Supply Chain Management under the Threat of International Terrorism,” *The International Journal of Logistics Management*, 12 (2001), 1-11.
- [15] Song, J. and P. Zipkin, “Inventory Control in a Fluctuating Demand Environment,” *Operations Research*, 41 (1993), 351-370.
- [16] Song, J. and P. Zipkin, “Inventory Control with Information About Supply Conditions,” *Management Science*, 42 (1996), 1409-1419.
- [17] Weiss, H. and C. Rosenthal, “Optimal Ordering Policies When Anticipating a Disruption in Supply or Demand,” *European Journal of Operational Research*, 59 (1992), 370-382.
- [18] World Trade Organization, “International Trade Statistics 2004,” World Trade Organization, 2004.