Problem 1
We want to calculate the following expected value (d-dimensional integral) of $f_d(\omega)$, where $\omega$ is uniformly distributed on the hypercube $[0,10]^d$:

$$E_d[f_d(\omega)] = \int_{[0,10]^d} \prod_{i=1}^{d} \left[ \frac{10}{\sqrt{\omega_i}} + \omega_i \sin(\omega_i^2) \right] \frac{1}{10^d} d\omega$$

Note that this integral decomposes into a separate integral for each dimension, and is easy to calculate exactly. Calculate the exact value of $E_d[f_d(\omega)]$. Now pretend that the integral does not decompose. The objective is to empirically compare the efficiency of different computational methods for approximating $E_d[f_d(\omega)]$. Use a multidimensional numerical integration method of your choice, such as the trapezoid method or Simpson’s method. Note: Do not simply calculate the one-dimensional integrals with a numerical integration method and then use the results. Although that would be correct for this particular integral because of its simple form, the point of the exercise is to compare the accuracy and efficiency of different methods for computing multidimensional integrals. Also use a random sampling method (Monte Carlo integration). Compute $E_d[f_d(\omega)]$ with these methods, for $d = 1, 2, \ldots$, (see how large you can make $d$ with less than 2 hours of computing time) and for varying number $n$ of points $\omega^j$ where the function $f_d(\omega^j)$ is evaluated. For each of these combinations of method, $d$, and $n$, compute the root square error (which you can do exactly, but which is random for the sampling method). For the sampling method, you may want to obtain a more accurate estimate of the root mean square error by replicating the experiment a number of times. Make plots of the root mean square error versus $n$, and compare the efficiency of the methods for different $d$. 
Problem 2
Consider the following optimization problem:

\[
\min_{x \in \{-1,0,1\}^3} \{ g(x) \equiv E[G(x, \omega)] \}
\]  

(1)

where \( \omega \in \Omega = \mathbb{R}^9 \),

\[
G(x, \omega) \equiv a(\omega)^T x + x^T B(\omega) x
\]

and

\[
a(\omega) \equiv (0.1 + \omega_1, 0.2 + \omega_2, 0.3 + \omega_3)^T
\]

and

\[
B(\omega) \equiv \begin{bmatrix}
1 + \omega_{11}, & 0.7 + \omega_{12}, & 0.8 + \omega_{13} \\
0.7 + \omega_{12}, & 1 + \omega_{22}, & 0.9 + \omega_{23} \\
0.8 + \omega_{13}, & 0.9 + \omega_{23}, & 1 + \omega_{33}
\end{bmatrix}
\]

Each \( \omega_i \) and \( \omega_{ij} \) has mean 0. Note that it is easy to calculate \( g(x) \) for each \( x \in \{-1,0,1\}^3 \), and thus it is easy to solve the optimization problem (1) by enumeration. Go ahead and solve the optimization problem (1). Now pretend that it is not that easy to calculate \( g(x) \) for each \( x \). Assume that the random variables \( \omega_i \) and \( \omega_{ij} \) are all independent, with each \( \omega_i \) uniformly distributed on \([-2, 2]\), and each \( \omega_{ij} \) uniformly distributed on \([-1, 1]\). Use a ranking and selection method to choose a solution. Specify the method that you use. Compare the method’s actual performance with the method’s predicted performance. Evaluate the sensitivity of the performance of the method with respect to deviations from the assumptions of the method. Evaluate the efficiency of the method.